Pimpalner Education Society's

Karm. A. M. Patil Arts, Commerce and Kai. Annasaheb N. K. Patil Science Senior College Pimpalner, Tal.- Sakri, Dist.- Dhule.

CLASS NOTES

CLASS: F.Y.B.SC SEM.-II

SUBJECT: MTH-202: THEORY OF EQUATIONS

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MTH 202: THEORY OF EQUATIONS

Unit-1. Divisibility of Integers **No. of Periods – 12**

Natural numbers. Well ordering principle (statement only). Principle of Mathematical Induction. Divisibility of integers and theorems. Division algorithm. GCD and LCM. Euclidean algorithm. Unique factorization theorem.

Unit-2. Polynomials **No. of Periods – 12**

Revision of Polynomials, Horner's method of synthetic division, Existence and uniqueness of GCD of two polynomials, Polynomial equations, Factor theorem and generalized factor theorem for polynomials, Fundamental theorem of algebra (Statement only), Methods to find common roots of polynomial equation, Descarte's rule of signs, Newton's method of divisors for the integral roots.

Unit-3. Theory of Equations-I **No. of Periods – 11**

Relation between roots and coefficient of general polynomial equation in one variable. Relation between roots and coefficient of quadratic, cubic and biquadratic equations. Symmetric functions of roots.

Unit-4. Theory of Equations –II **No. of Periods – 10**

Transformation of equations. Cardon's method of solving cubic equations. Biquadratic equations. Descarte's method of solving biquadratic equations.

Reference Books:

1. Elementary Number Theory, by David M. Burton, W. C. Brown publishers, Dubuquo lowa 1989.
2. Higher Algebra, by H. S. Hall and S. R. Knight, H. M. Publications 1994.
3. Matrix and Linear Algebra, by K. B. Datta, Prentice Hall of India Pvt. Ltd. New Delhi, 2000.
4. Theory of Equations, by D. R. Sharma, Sharma Publications, Jalandar.

Learning Outcomes:

Students can find out roots of any equation of degree less than or equal to five. Theory of equations is highly useful in various subjects like algebra, linear algebra, calculus, ordinary and partial differential equations etc.

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UNIT-1. DIVISIBILITY OF INTEGERS

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Natural numbers: The numbers used for counting are called natural numbers and the set of natural numbers is denoted by N = {1, 2, 3, 4, … … }

Remark: i) W = {0, 1, 2, 3, 4, … … } is called set of whole numbers.

ii) I or Z = { … … , -3, -2, -1, 0, 1, 2, 3, … … } is called set of integers.

Properties of addition and multiplication: For any three numbers a, b, c,

i) $a + (b + c) = (a + b) + c$ (Associative law w.r.t. addition)

ii) $a.(b.c) = (a.b).c$ (Associative law w.r.t. multiplication)

iii) $a + b = b + a$ (Commutative law w.r.t. addition)

iv) $a.b = b.a$ (Commutative law w.r.t. multiplication)

v) $a + 0 = 0 + a = a$ (Law of identity w.r.t. addition)

vi) $a.1 = 1.a = a$ (Law of identity w.r.t. multiplication)

vii) $a.(b + c) = a.b + a.c$ (Left distributive law)

viii) $(b + c).a = b.a + c.a$ (Right distributive law)

Remark: i) All the above laws are satisfied by W and Z.

ii) All the above laws are satisfied by N except $a + 0 = 0 + a = a$

Order Relation in N: Let m and n ∈ N, then m is said to be less than n written as $m < n$ if there exists $p \in N$ such that $m + p = n$

e.g. i) $5 < 8$ because there exists $3 \in N$ such that $5 + 3 = 8$

ii) $2 < 6$ because there exists $4 \in N$ such that $2 + 4 = 6$

Law of Trichotomy:

For any two natural numbers m and n, only one of the following holds

a) $m = n$ b) $m < n$ c) $n < m$.

Transitive Property: For m, n, p ∈ N, if $m < n$ and $n < p$ then $m < p$

Note: i) If $m < n$ then $m + p < n + p \ \forall\, m, n, p \in N$

ii) If $m < n$ then $mp < np \ \forall\, m, n, p \in N$

Well ordering principle: Every nonempty subset of N has a least element.

Principle of Mathematical Inductions:

First Principle of Finite Induction:

Let P(n) be the statement for n ∈ N, such that

i) P(1) is true.

ii) P(k) is true ⇒ P(k+1) is true $\forall\, k \geq 1$

Then P(n) is true for all n ∈ N.

Generalized form of the First Principle of Finite Induction:

Let P(n) be the statement for n ∈ N, such that

i) P(k) is true for some fixed k ∈ N.

ii) P(m) is true ⇒ P(m+1) is true $\forall\, m \geq k$

Then P(n) is true $\forall\, n \geq k$.

Second Principle of Finite Induction:

Let P(n) be the statement for $n \in N$, such that

i) P(1) is true.

ii) P(r) is true $\forall\, r < m \Rightarrow$ P(m) is true

Then P(n) is true for all $n \in N$.

Generalized form of the Second Principle of Finite Induction:

Let P(n) be the statement for $n \in N$, such that

i) P(k) is true for some fixed $k \in N$.

ii) P(r) is true $\forall\, k \le r < m \Rightarrow$ P(m) is true $\forall\, m \ge k$

Then P(n) is true $\forall\, n \ge k$.

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Ex. Prove by method of induction that the sum of all natural numbers is $\frac{n(n+1)}{2}$

Or prove that $1+2+3+ \ldots\ldots + n = \frac{n(n+1)}{2}$

Proof: Let P(n) : $1+2+3+ \ldots\ldots + n = \frac{n(n+1)}{2}$ be the statement for $n \in N$.

Step-i) For n = 1, we have, LHS = 1 and RHS = $\frac{1(1+1)}{2} = 1$

$\therefore$ LHS = RHS

$\therefore$ P(1) is true.

Step-ii) Suppose P(k) is true

i.e. $1+2+3+ \ldots\ldots + k = \frac{k(k+1)}{2}$ …… (1)

Adding both sides by (k+1), we get

$$1+2+3+ \ldots\ldots + k+(k+1) = \frac{k(k+1)}{2} + (k+1)$$

$$= \frac{k(k+1)+2(k+1)}{2}$$

$$= \frac{(k+1)(k+2)}{2}$$

$\therefore$ P(k+1) is true.

i.e. P(k) is true $\Rightarrow$ P(k+1) is true $\forall\, k \ge 1$

$\therefore$ By first principle of induction, P(n) is true $\forall\, n \in N$.

i.e. $1+2+3+ \ldots\ldots + n = \frac{n(n+1)}{2}\ \forall\, n \in N$

Hence proved.

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Ex. Prove that the sum of all first n odd natural numbers is n^2.

Or prove that $1+3+5+ \ldots\ldots + (2n-1) = n^2$.

Proof: Let P(n) : $1+3+5+ \ldots\ldots + (2n-1) = n^2$ be the statement for $n \in N$.

Step-i) For n = 1, we have, LHS = 1 and RHS = $1^2 = 1$

$\therefore$ LHS = RHS

$\therefore$ P(1) is true.

Step-ii) Suppose P(k) is true

i.e. $1+3+5+ \ldots\ldots + (2k-1) = k^2$ …… (1)

Adding both sides by next odd number 2(k+1)-1, we get

$1+3+5+ \ldots\ldots + (2k-1)+[2(k+1)-1] = k^2+[2(k+1)-1]$

$= k^2+2k+1$

$= (k+1)^2$

∴ P(k+1) is true.

i.e. P(k) is true ⇒ P(k+1) is true $\forall\, k \geq 1$

∴ By first principle of induction, P(n) is true $\forall\, n \in N$.

i.e. $1+3+5+ \ldots\ldots + (2n -1) = n^2 \;\forall\, n \in N$

Hence proved.

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Ex. Prove that $1^2+2^2+3^2+ \ldots\ldots + n^2 = \frac{n(n+1)(2n+1)}{6} \;\forall\, n \in N$

Proof: Let P(n) : $1^2+2^2+3^2+ \ldots\ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$ be the statement for $n \in N$.

Step-i) For n = 1, we have, LHS = $1^2 = 1$ and RHS = $\frac{1(1+1)(2+1)}{6} = 1$

∴ LHS = RHS

∴ P(1) is true.

Step-ii) Suppose P(k) is true

i.e. $1^2+2^2+3^2+ \ldots\ldots + k^2 = \frac{k(k+1)(2k+1)}{6}$ …… (1)

Adding both sides next term $(k+1)^2$, we get

$1^2+2^2+3^2+ \ldots\ldots + k^2+(k+1)^2 = \frac{k(k+1)(2k+1)}{6} +(k+1)^2$

$= \frac{k(k+1)(2k+1)+6(k+1)^2}{6}$

$= \frac{(k+1)(2k^2+k+6k+6)}{6}$

$= \frac{(k+1)(2k^2+7k+6)}{6}$

$= \frac{(k+1)(k+2)(2k+3)}{6}$

∴ P(k+1) is true.

i.e. P(k) is true ⇒ P(k+1) is true $\forall\, k \geq 1$

∴ By first principle of induction, P(n) is true $\forall\, n \in N$.

i.e. $1^2+2^2+3^2+ \ldots\ldots + n^2 = \frac{n(n+1)(2n+1)}{6} \quad \forall\, n \in N$

Hence proved.

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Ex. Use principle of induction to prove that $5^n + 3$ is divisible by 4.

Proof: Let P(n) : $5^n + 3$ is divisible by 4 be the statement for $n \in N$.

Step-i) For n = 1, we have, $5^1 + 3 = 8$ which is divisible by 4.

$\therefore$ P(1) is true.

Step-ii) Suppose P(k) is true

i.e.$5^k + 3$ is divisible by 4

$\therefore 5^k + 3 = 4r$ for some $r \in Z$ …… (1)

Consider

$5^{k+1} + 3 = 5.5^k + 3$

$= 5(4r - 3) + 3$ by(1)

$= 20r - 15 + 3$

$= 20r - 12$

$\therefore 5^{k+1} + 3 = 4(5r - 3)$ is divisible by 4.

$\therefore$ P(k+1) is true.

i.e. P(k) is true $\Rightarrow$ P(k+1) is true $\forall\, k \geq 1$

$\therefore$ By first principle of induction, P(n) is true $\forall\, n \in N$.

i.e. $5^n + 3$ is divisible by 4. $\forall\, n \in N$

Hence proved.

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Ex. Show by principle of induction '$7^n + 2$ is divisible by 3'

Proof: Let P(n) : '$7^n + 2$ is divisible by 3' be the statement for $n \in N$.

Step-i) For n = 1, we have, $7^1 + 2 = 9$ which is divisible by 3.

$\therefore$ P(1) is true.

Step-ii) Suppose P(k) is true

i.e.$7^k + 2$ is divisible by 3

$\therefore 7^k + 2 = 3r$ for some $r \in Z$ …… (1)

Consider

$7^{k+1} + 2 = 7.7^k + 2$

$= 7(3r - 2) + 2$ by(1)

$= 21r - 14 + 2$

$= 21r - 12$

$\therefore 7^{k+1} + 2 = 3(7r - 4)$ is divisible by 3.

$\therefore$ P(k+1) is true.

i.e. P(k) is true $\Rightarrow$ P(k+1) is true $\forall\, k \geq 1$

$\therefore$ By first principle of induction, P(n) is true $\forall\, n \in N$.

i.e. $7^n + 2$ is divisible by 3. $\forall\, n \in N$

Hence proved.

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Ex. Prove that $2^n < n!\ \forall\, n \geq 4$.

Proof: Let P(n): $2^n < n!\ \forall\, n \geq 4$

Step-i) For n = 4, we have, $2^4 = 16$ and $4! = 24$

$\therefore 2^4 < 4!$

$\therefore$ P(4) is true.

Step-ii) Suppose P(k) is true

i.e. $2^k < k!$ …… (1) for $k \geq 4$

As $4 \leq k \Rightarrow 2 < k < k+1$

$\therefore 2^k . 2 < (k!)(k+1)$

$\therefore 2^{k+1} < (k+1)!$

$\therefore$ P(k+1) is true.

i.e. P(k) is true $\Rightarrow$ P(k+1) is true $\forall\ k \geq 4$

$\therefore$ By generlized form of first principle of induction,

P(n) is true $\forall\ n \geq 4$.

i.e. $2^n < n!\ \forall\ n \geq 4$

Hence proved.

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Divisibility of Integers: Let $a, b \in Z$, $a \neq 0$. If there exist $c \in Z$ such that $b = ac$, then it is said that 'a divides b' and denoted by $a|b$.

e.g. i) $3|15 \because 15 = 3x5$, i) $7|(-28) \because -28 = 7x(-4)$

Remark: i) $a|b$ is read as a divides b or b is multiple of a or a is divisor of b or b is divisible by a or a is factor of b.

ii) If $a \neq 0$, then $a|0$ and $a|(\pm a)$

iii) a does not divide b is written as $a \nmid b$

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Theorem: $a|b$ and $b|c$ then $a|c$

Proof: Let $a|b$ and $b|c$

$\Rightarrow b = ar$ and $c = bk$ for some $r, k \in Z$

$\Rightarrow c = (ar)k$ by putting value of b.

$\Rightarrow c = a(rk)$ where $rk \in Z$

$\Rightarrow a|c$

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Theorem: $a|b$ and $a|c$ then $a|b\pm c$

Proof: Let $a|b$ and $a|c$

$\Rightarrow b = ar$ and $c = ak$ for some $r, k \in Z$

$\Rightarrow b\pm c = ar\pm ak$

$\Rightarrow b\pm c = a(r\pm k)$ where $(r\pm k) \in Z$

$\Rightarrow a|b\pm c$

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Theorem: $a|b$ and $a|c$ then $a|bx + cy$ for all $x, y \in Z$

Proof: Let $a|b$ and $a|c$

$\Rightarrow b = ar$ and $c = ak$ for some $r, k \in Z$

$\Rightarrow bx+cy = (ar)x+(ak)y$ where $x, y \in Z$

$\Rightarrow bx+cy = a(rx+ky)$ where $rx+ky \in Z$

$\Rightarrow a|bx+cy$

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Theorem: $a|b$ then $a|bc\ \forall\ c \in Z$

Proof: Let $a|b$

$\Rightarrow b = ar$ for some $r \in Z$

$\Rightarrow bc = (ar)c$ where $c \in Z$

$\Rightarrow bc = a(rc)$ where $rc \in Z$

$\Rightarrow a|bc$

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Theorem: $a|b$ and $b|a$ then $a = \pm b$

Proof: Let $a|b$ and $b|a$

$\Rightarrow b = ar$ and $a = bk$ for some $r, k \in Z$

$\Rightarrow b = (bk)r$

$\Rightarrow b = b(kr)$

$\Rightarrow kr = 1$

$\Rightarrow k = r = \pm 1 \quad \because r, k \in Z$

$\Rightarrow a = \pm b$

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Division Algorithm: If a and b are any two integers and $b \neq 0$ then there exist unique integers q and r such that $a = bq + r$ where $0 \leq r < |b|$

e.g. i) For 7 and 50, we have $50 = 7x7+1$, ii) For 9 and 80, we have $80 = 9x8 + 8$

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Greatest Common Divisor (GCD): Let a and b be any two non-zero integers then the positive integers d is called greatest common divisor (GCD) of a and b if i) $d|a$ and $d|b$, ii) if $c|a$ and $c|b$ then $c|d$. Denoted by $d = (a, b)$.

e.g. i) $(12, -18) = 6$, ii) $(75, 48) = 3$

Least Common Multiple (LCM): Let a and b be any two non-zero integers then the positive integers l is called least common multiple (LCM) of a and b if i) $a|l$ and $b|l$, ii) if $a|c$ and $b|c$ then $l|c$. Denoted by $l = [a, b]$.

e.g. i) $[6, 10] = 30$, ii) $[75, 48] = 1200$

Note: $a\times b = (a, b)\times[a, b]$

Prime Number: A non-zero integer $a\neq 1$ is called a prime number if it is divisible by ± 1 and $\pm a$ only.

e.g. 2, 3, 5 etc. are prime numbers.

Composite Number: A non-zero integer a is called a composite number if it is product of two or more prime numbers.

e.g. 4, 6 and 15 are composite numbers.

Relatively Prime Integers: Two non-zero integer a and b are said to be relatively prime integers if (a, b) = 1.

e.g. 4 and 15 are relatively prime integers ∵ (4, 15) = 1.

Remark: Two distinct prime numbers are always relatively prime integers. But if (a, b) = 1 then a and b may or may not be prime.

Euclidean Algorithm: The process of finding g.c.d. of two integers by applying division algorithm repeatedly is called Euclidean algorithm.

Remark: g.c.d. of any two integers is expressed into linear form of them. i.e. if (a, b) = d then there exists some integers m and n such that d = ma + nb.

Unique factorization theorem: Every positive integer a > 1 is uniquely expressed as the product of primes irrespective of their orders.

e.g. i) 12 = 2×2×3 or 2×3×2 or 3×2×2, ii) 28 = 2×2×7 or 2×7×2 or 7×2×2.

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Ex. Find g.c.d. of 75 and 48. Also express in the form (75, 48) = 75m+48n

Solution: By Euclidean algorithm, we get

$$75 = 1\times 48 + 27 \quad \dots (1)$$
$$48 = 1\times 27 + 21 \quad \dots (2)$$
$$27 = 1\times 21 + 6 \quad \dots (3)$$
$$21 = 3\times 6 + 3 \quad \dots (4)$$
$$6 = 2\times 3 + 0$$

∴ (75, 48) = 3

Now from (4), we get

$$3 = 21 - 3\times 6$$
$$= 21 - 3\times(27 - 1\times 21) \quad \text{by (3)}$$
$$= 4\times 21 - 3\times 27$$
$$= 4\times(48 - 1\times 27) - 3\times 27 \quad \text{by (2)}$$
$$= 4\times 48 - 7\times 27$$
$$= 4\times 48 - 7\times(75 - 1\times 48) \quad \text{by (1)}$$
$$= 11\times 48 - 7\times 75$$
$$3 = 75(-7) + 48(11)$$

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Ex. Find g.c.d. of 483 and 574, and express g.c.d ma + nb

Solution: By Euclidean algorithm, we get

$$574 = 1\times 483 + 91 \quad \dots (1)$$
$$483 = 5\times 91 + 28 \quad \dots (2)$$
$$91 = 3\times 28 + 7 \quad \dots (3)$$
$$28 = 4\times 7 + 0$$

∴ (483, 574) = 7

Now from (3), we get

$$7 = 91 - 3\times 28$$
$$= 91 - 3\times(483 - 5\times 91) \qquad \text{by (2)}$$
$$= 16\times 91 - 3\times 483$$
$$= 16\times(574 - 1\times 483) - 3\times 483 \qquad \text{by (1)}$$
$$= 16\times 574 - 19\times 483$$
$$7 = 483(-19) + 574(16)$$

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Ex. If a, b, m, n are non-zero integers such that ma + nb= 1, then show that

$$(a, b) = (m, n) = (a, n) = (m, b) = 1$$

Proof: Let (a, b) = d i.e. g.c.d. of a and b is d.

$\therefore$ d|a and d|b $\Rightarrow$ a = dr and b = dk for some r, k $\in$ Z

$\therefore$ ma + nb= 1 gives

m(dr) + n(dk) = 1

$\therefore$ d(mr+nk) = 1

$\therefore$ d|1 $\quad\because$ mr+nk $\in$ Z

$\therefore$ d = 1 $\quad\because$ d > 0

$\therefore$ (a, b) = 1

Similarly we can prove (m, n) = (a, n) = (m, b) = 1.

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Ex. If d = (a, b), a = dx , b = dy; x, y $\in$ Z, then show that (x, y) = 1.

Proof: Let (a, b) = d

$\therefore$ ma + nb= d for some m, n $\in$ Z

$\therefore$ m(dx) + n(dy) = d $\quad\because$a = dx , b = dy; x, y $\in$ Z

$\therefore$ d(mx+ny) = d

$\therefore$ mx+ny = 1

$\therefore$ (x, y) = 1 $\quad$ Hence proved.

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Ex. If d = (a, b), a|c, b|c, then show that ab|cd.

Proof: Let (a, b) = d

$\therefore$ ma + nb= d for some m, n $\in$ Z $\quad$ … (1)

Also a|c , b|c $\Rightarrow$ c = ar and c = bk for some r, k $\in$ Z

Multiplying by c to (1), we get.

$\therefore$ mac + nbc= cd

$\therefore$ ma(bk) + nb(ar)= cd

$\therefore$ ab(mk+nr) = cd

$\therefore$ ab|cd $\quad\because$mk+nr $\in$ Z

Hence proved.

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Euclid's Lemma. If p is prime and a and b are integers such that p|ab then either p|a or p|b

Proof: Let p is prime and a and b are integers such that p|ab.

If p|a, then we are through.

If p∤a, then (a, p) = 1

$\therefore$ ma + np= 1 for some m, n $\in$ Z … (1)

Multiplying by b to (1), we get.

$\therefore$ mab + npb= b

$\therefore$ m(pk) + npb= b $\because$ p|ab $\Rightarrow$ ab = pk for some k $\in$ Z

$\therefore$ p(mk+nb) = b

$\therefore$ p|b $\because$mk+nb $\in$ Z

Hence if p|ab then either p|a or p|b is proved.

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Ex. Show that $\sqrt{5}$ is not a rational number.

Proof: Suppose $\sqrt{5}$ is a rational number.

$\therefore \sqrt{5} = \frac{p}{q}$ where p, q $\in$ Z and (p, q) = 1

$\therefore$ p = $\sqrt{5}$ q

$\therefore p^2 = 5q^2$ …… (1)

$\therefore 5|p^2 \Rightarrow 5|p$ $\because$ 5 is prime.

$\therefore$ p = 5k for some k $\in$ Z

$\therefore p^2 = 25k^2$

$\therefore 5q^2 = 25k^2$ by (1)

$\therefore q^2 = 5k^2$

$\therefore 5|q^2 \Rightarrow 5|q$ $\because$ 5 is prime.

Now 5|p and 5|q $\Rightarrow$(p, q) $\geq$ 5 which contradicts to (p, q) = 1

$\therefore$ Our assumption is wrong.

$\therefore \sqrt{5}$ is not a rational number.

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Ex. Show that $\sqrt{7}$ is not a rational number.

Proof: Suppose $\sqrt{7}$ is a rational number.

$\therefore \sqrt{7} = \frac{p}{q}$ where p, q $\in$ Z and (p, q) = 1

$\therefore$ p = $\sqrt{7}$ q

$\therefore p^2 = 7q^2$ …… (1)

$\therefore 7|p^2 \Rightarrow 7|p$ $\because$ 7 is prime.

$\therefore$ p = 7k for some k $\in$ Z

$\therefore p^2 = 49k^2$

$\therefore 7q^2 = 49k^2$ by (1)

$\therefore q^2 = 7k^2$

$\therefore 7|q^2 \Rightarrow 7|q$ $\because$ 7 is prime.

Now $7|p$ and $7|q \Rightarrow (p, q) \geq 7$ which contradicts to $(p, q) = 1$

$\therefore$ Our assumption is wrong.

$\therefore \sqrt{7}$ is not a rational number.

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MULTIPLE CHOICE QUESTIONS

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1) For $m, n, p \in N$, if $m < n$ and $n < p$ then ……

[A] $m < p$ **[B]** $m > p$ **[C]** $m = p$ **[D]** $n < m$

2) Every nonempty subset of N has a …… element

[A] 0 **[B]** greatest **[C]** least **[D]** 1

3) Let P(n) be the statement for $n \in N$, such that

i) P(1) is true. ii) P(k) is true $\Rightarrow$ P(k+1) is true $\forall\, k \geq 1$

Then P(n) is true for all $n \in N$. is the statement of ……

[A] first principle of finite induction

[B] second principle of finite induction

[C] generalized form of first principle of finite induction

[D] None of These

4) Let P(n) be the statement for $n \in N$, such that

i) P(1) is true, ii) P(r) is true $\forall\, r < m \Rightarrow$ P(m) is true

Then P(n) is true for all $n \in N$ is the statement of ……

[A] first principle of finite induction

[B] second principle of finite induction

[C] generalized form of first principle of finite induction

[D] None of These

5) $1+2+3+ \ldots\ldots + n = \ldots\ldots$

[A] $\frac{n(n+1)(2n+1)}{6}$ **[B]** n^2 **[C]** $\frac{n(n+1)}{2}$ **[D]** $\frac{n(n-1)}{2}$

6) $1+3+5+ \ldots\ldots +(2n-1) = \ldots\ldots$

[A] $\frac{n(n+1)(2n+1)}{6}$ **[B]** n^2 **[C]** $\frac{n(n+1)}{2}$ **[D]** $\frac{n(n-1)}{2}$

7) $1^2+2^2+3^2+ \ldots\ldots + n^2 = \ldots\ldots$

[A] $\frac{n(n+1)(2n+1)}{6}$ **[B]** n^2 **[C]** $\frac{n(n+1)}{2}$ **[D]** $\frac{n(n-1)}{2}$

8) a|b is read as ……

[A] a divides b **[B]** b is multiple of a

[C] b is divisible by a **[D]** All of These

9) For any natural number n, $5^n + 3$ is divisible by ……

[A] 3 **[B]** 4 **[C]** 5 **[D]** 6

10) For any natural number n, $7^n + 2$ is divisible by ……

[A] 3 **[B]** 4 **[C] 5** **[D]** 8

11) $2^n <$ n! for all …… , n € N

[A] n < 4 **[B]** n **[C]** n ≥ 4 **[D]** n ≥ 2

12) If a|b and b|c then ……

[A] b|a **[B]** c|a **[C]** a|c **[D]** c|b

13) If a|b and a|c then ……

[A] a|bc **[B]** a|b±c **[C]** a|bx+cy **[D]** All of These

14) If a|b and b|a then a = ……

[A] 1 **[B]** ±b **[C] 0** **[D]** 2

15) If a ≠ 0, then a|….

[A] 0 only **[B]** a only **[C]** 0 and ±a **[D]** any nmber

16) If a and b any two integers with b ≠ 0 then there exist unique integers q and r such that a = …… where 0 ≤ r < |b|

[A] bq+r **[B]** bq-r **[C]** bq **[D]** r

17) g.c.d of 12 and 15 is ……

[A] 3 **[B]** 12 **[C]** 15 **[D]** 6

18) g.c.d of 75 and 48 is ……

[A] 3 **[B]**6 **[C]** 4 **[D]** 12

19) (12, -18) = ……

[A] 3 **[B]** 4 **[C]** 6 **[D]** 18

20) L.C.M of 6 and 10 is ……

[A] 6 **[B]** 10 **[C]** 30 **[D]** 15

21) If g.c.d. and l.c.m. of integers a and b are d and *l* respectively then ab = ……

[A] d*l* **[B]** d **[C]** *l* **[D]** d+*l*

22) If (75, 48) = 3, then [75, 48] = ……

[A] 1200 **[B]** 600 **[C]** 300 **[D]** 900

23) If a and b are relatively prime then g.c.d of a and b is ……

[A] 0 **[B]** 1 **[C]** -1 **[D]** 2

24) If a and b both are prime then g.c.d of a and b is ……

[A] a **[B]** b **[C]** 1 **[D]** 2

25) If a and b are relatively prime then a and b are ……

[A] both prime **[B]** both not prime

[C] may or may not be prime **[D]** None of These

26) If a, b, m, n are non-zero integers such that ma + nb= 1, then (a, b) = (m, n) = (a, n) = (m, b) = …

[A] 1 **[B]** -1 **[C]**0 **[D]** a

27) If (a, k) = (b, k) = 1 then (ab, k) = ……

[A] 0 **[B]** 1 **[C]** -1 **[D]** k

28) If p is prime and p|ab then ……

[A] p|a **[B]** p|b **[C]** p|a or p|b **[D]** None of These

29) If (a, b) = d , a|c , b|c then ……

[A] ab|cd **[B]** c|d **[C]** a|b **[D]** c|a

30) If (a, b) = 1 then (a^2, b^2) = ……

[A] 1 **[B]** 0 **[C]** -1 **[D]** 2

31) For an even integer n, x^n-y^n is divisible by ……

[A] only (x-y) **[B]** only (x+y)

[C] both (x-y) and (x+y) **[D]** None of These

32) For an odd integer n, x^n-y^n is divisible by ……

[A] only (x-y) **[B]** only (x+y)

[C] both (x-y) and (x+y) **[D]** None of These

33) $\sqrt{3}$, $\sqrt{5}$ and $\sqrt{7}$ are ……

[A] rational numbers **[B]** not rational numbers

[C] integers **[D]** None of These

UNIT-2. POLYNOMIALS

==

Polynomials: $f(x) = a_0+a_1x+a_2x^2+\ldots\ldots+a_nx^n$ with $a_n \neq 0$ is called a polynomial of degree n.

Monic Polynomials: $f(x) = a_0+a_1x+a_2x^2+\ldots\ldots+a_nx^n$ with $a_n = 1$ is called a monic polynomial of degree n.

e.g. $x^5 + x^4 + x^3 + x^2 + x$ is a monic polynomial.

An Algebraic Equation: $f(x) = a_0+a_1x+a_2x^2+\ldots\ldots+a_nx^n = 0$ with $a_n \neq 0$ is called an algebraic equation of degree n.

Monic Polynomials Equation: $f(x) = a_0+a_1x+a_2x^2+\ldots\ldots+a_nx^n = 0$ with $a_n = 1$ is called a monic polynomial equation of degree n.

e.g. $x^5 + 4x^3 + 2 = 0$ is a monic polynomial equation of degree 5.

Root: x is called root of an equation $f(x) = 0$ if it satisfies the equation $f(x) = 0$.

Note: Q[x] = The set of all polynomials with rational coefficients.

R[x] = The set of all polynomials with real coefficients.

C[x] = The set of all polynomials with complex coefficients.

Remark: i) The polynomial of degree 1 is called linear polynomial.

ii) The polynomial of degree 2 is called quadratic polynomial.

iii) The polynomial of degree 3 is called cubic polynomial.

iv) The polynomial of degree 4 is called biquadratic polynomial.

v) An equation of degree 1 is called linear equation.

vi) An equation of degree 2 is called quadratic equation.

vii) An equation of degree 3 is called cubic equation.

viii) An equation of degree 4 is called biquadratic equation.

ix) Root of an equation $f(x) = 0$ may be real or complex.

Equality: Two polynomials $f(x) = a_0+a_1x+a_2x^2+\ldots\ldots+a_mx^m$ and $g(x) = b_0+b_1x+b_2x^2+\ldots\ldots+b_nx^n$ in Q[x] are said to be equal if $a_i = b_i$ for all i and $m = n$.

Addition: Let $f(x) = a_0+a_1x+a_2x^2+\ldots\ldots+a_mx^m$ & $g(x) = b_0+b_1x+b_2x^2+\ldots\ldots+b_nx^n$ be any two polynomials in Q[x], then their sum $f(x)+g(x)$ is defined by

$f(x)+g(x) = (a_0+ b_0)+(a_1+ b_1)x+(a_2+b_2)x^2+\ldots\ldots$

Multiplication: Let $f(x) = a_0+a_1x+a_2x^2+\ldots\ldots+a_mx^m$ and $g(x) = b_0+b_1x+b_2x^2+\ldots\ldots+b_nx^n$ be any two polynomials in Q[x], then their multiplication $f(x)g(x)$ is defined by $f(x)g(x) = c_0+c_1x+c_2x^2+\ldots\ldots+c_{m+n}x^{m+n}$

where $c_0 = a_0b_0$, $c_1 = a_0b_1+ a_1b_0$, $c_2 = a_0b_2+ a_1b_1+ a_2b_0$, ……

$c_i = a_0b_i+ a_1b_{i-1}+ a_2b_{i-2} + \ldots\ldots+ a_ib_0$, ……

Remark: If degree of $f(x) = m$ and degree of $g(x) = n$, then

degree of $f(x)+g(x) = \max.\{m, n\}$ and degree of $f(x)g(x) = m + n$

==

Ex. If $f(x) = 3+5x-7x^2+\frac{1}{2}x^3$ and $g(x) = 5+7x-5x^2+2x^3+9x^4$,

then find i) f(x)+g(x) and ii) f(x)g(x)

Solution: Let $f(x) = 3+5x-7x^2+\frac{1}{2}x^3$ and $g(x) = 5+7x-5x^2+2x^3+9x^4$,

$\therefore$ i) $f(x)+g(x) = 3+5x-7x^2+\frac{1}{2}x^3+5+7x-5x^2+2x^3+9x^4$

$= (3+5)+(5+7)x+(-7-5)x^2+(\frac{1}{2}+2)x^3+9x^4$

$= 8 + 12x - 12x^2 + \frac{5}{2}x^3 + 9x^4$

and ii) $f(x)g(x) = (3+5x-7x^2+\frac{1}{2}x^3)(5+7x-5x^2+2x^3+9x^4)$

$= 15+21x-15x^2+6x^3+27x^4+25x+35x^2-25x^3+10x^4+45x^5$

$-35x^2-49x^3+35x^4 -14x^5- 63x^6+\frac{5}{2}x^3+\frac{7}{2}x^4-\frac{5}{2}x^5+x^6+\frac{9}{2}x^7$

$= 15+(21+25)x+(-15+35-35)x^2+(6-25-49+\frac{5}{2})x^3$

$+ (27+10+35+\frac{7}{2})x^4+(45-14-\frac{5}{2})x^5+(-63+1)x^6+\frac{9}{2}x^7$

$= 15+46x-15x^2-\frac{131}{2}x^3+\frac{151}{2}x^4+\frac{57}{2}x^5-62x^6+\frac{9}{2}x^7$

==

Ex. If $f(x) = 1-3x+2x^2$ and $g(x) = 3 - \frac{1}{2}x+4x^2 - x^3$,

then find i) f(x)+g(x) and ii) f(x)g(x)

Solution: Let $f(x) = 1-3x+2x^2$ and $g(x) = 3 - \frac{1}{2}x+4x^2 - x^3$,

$\therefore$ i) $f(x)+g(x) = 1-3x+2x^2+3 - \frac{1}{2}x+4x^2 - x^3$

$= (1+3)+(-3-\frac{1}{2})x+(2+4)x^2 - x^3$

$= 4 - \frac{7}{2}x + 6x^2 - x^3$

and ii) $f(x)g(x) = (1-3x+2x^2)(3 - \frac{1}{2}x+4x^2 - x^3)$

$= 3 - \frac{1}{2}x+4x^2 - x^3-9x+ \frac{3}{2}x^2-12x^3+3x^4+ 6x^2-x^3+8x^4-2x^5$

$= 3+(c-9)x+(4+\frac{3}{2}+6)x^2+(-1-12-1)x^3+(3+8)x^4-2x^5$

$= 3-\frac{19}{2}x+\frac{23}{2}x^2-14x^3+11x^4-2x^5$

==

Ex. Find the sum and product of f(x) and g(x), where

$f(x) = x^5 + 8x^4 - 2x^3 - 2x^2 - 16x + 4$ and $g(x) = x^4 + 7x^3 - 9x^2 + 10x - 2$,

then find i) f(x)+g(x) and ii) f(x)g(x)

Solution: Let $f(x) = x^5 + 8x^4- 2x^3 - 2x^2 -16x+ 4$ and $g(x) = x^4 +7x^3 - 9x^2 +10x - 2$,

$\therefore$ i) $f(x)+g(x) = x^5 + 8x^4 - 2x^3 - 2x^2 - 16x + 4 + x^4 + 7x^3 - 9x^2 + 10x - 2$

$= x^5 + (8+1)x^4 + (-2+7)x^3 + (-2-9)x^2 + (-16+10)x + (4-2)$

$= x^5 + 9x^4 + 5x^3 -11x^2 - 6x + 2$

and ii) $f(x)g(x) = (x^5 + 8x^4 - 2x^3 - 2x^2 - 16x + 4)\,(x^4 + 7x^3 - 9x^2 + 10x - 2)$

$= x^9 + 7x^8 - 9x^7 + 10x^6 - 2x^5 + 8x^8 + 56x^7 - 72x^6 + 80x^5 -16x^4$

$- 2x^7 -14x^6 +18x^5 - 20x^4 + 4x^3 - 2x^6 -14x^5 + 18x^4 - 20x^3 + 4x^2$

$-16x^5 -112x^4 +144x^3 -160x^2 +32x+ 4x^4 + 28x^3- 36x^2 + 40x-8$

$= x^9+(7+8)x^8+(-9+56-2)x^7+(10-72-14-2)x^6$

$+(-2+80+18-14-16)x^5+ (-16-20+18-112+4)x^4$

$+ (4-20+144+28)x^3 + (4-160-36)x^2 + (32+40)x – 8$

$= x^9 +15x^8 +45x^7 -78x^6 +66x^5 -126x^4 +156x^3 -192x^2 +72x-8$

==

Synthetic Division: If a polynomial of degree n is divided by linear term x+a. Then quotient will be a polynomial of degree n-1 and remainder will be constant. this process is called synthetic division.

==

Ex. Using Horner's method of synthetic division find quotient and remainder when $x^5 + 3x^4 + 5x^2 –2$ is divided by $x – 3$.

Solution: To find quotient and remainder when $x^5 + 3x^4 + 5x^2 –2$ i.e. $x^5 + 3x^4 + 0x^3 + 5x^2 + 0x –2$ is divided by $x – 3$, we use Horner's method of synthetic division as follows:

3	1	3	0	5	0	-2
		3	18	54	177	531
	1	6	18	59	177	529

∴ Quotient is $q(x) = x^4 + 6x^3 + 18x^2 + 59x + 177$ and remainder $r = 529$.

==

Ex. Use synthetic division to find quotient and remainder when $4x^3 + x^2 - 2x + 5$ is divided by $x – 3$.

Solution: To find quotient and remainder when $4x^3 + x^2 - 2x + 5$ is divided by $x – 3$, we use synthetic division as follows:

3	4	1	-2	5
		12	39	111
	4	13	37	116

∴ Quotient is $q(x) = 4x^2 + 13x + 37$ and remainder $r = 116$.

==

Ex. Use synthetic division to find quotient and remainder when $x^4 + x^3 + 4x^2 – x - 5$ is divided by $x – 1$.

Solution: To find quotient and remainder when $x^4 + x^3 + 4x^2 – x - 5$ is divided by $x – 1$, we use synthetic division as follows:

1	1	1	4	-1	-5
		1	2	6	5
	1	2	6	5	0

∴ Quotient is $q(x) = x^3 + 2x^2 + 6x + 5$ and remainder $r = 0$.

==

Ex. Express the polynomial $2x^3 + 3x + 2$ in powers of $x - 3$.
Hence find f(x-3) in powers of x.

Solution: To express the polynomial $2x^3 + 3x + 2$ i.e. $2x^3 + ox^2 + 3x + 2$ in powers of $x - 3$, we use synthetic division as follows:

3 |	2	0	3	2
|		6	18	63
|	2	6	21	65
|		6	36	
|	2	12	57	
|		6		
|	2	18		
|				
|	2			

$\therefore f(x) = 2(x-3)^3 + 18(x-3)^2 + 57(x-3) + 65$ which is in powers of (x-3).
Replacing x by x+3, we get,
$f(x+3) = 2x^3 + 18x^2 + 57x + 65$

===

Ex. Express the polynomial $2x^4 - 10x^3 + 3x + 4$ in powers of $x - 3$.
Hence find f(x+3) in powers of x.

Solution: To express the polynomial $2x^4 - 10x^3 + 3x + 4$ i.e. $2x^4 - 10x^3 + 0x^2 + 3x + 4$ in powers of $x - 3$, we use synthetic division as follows:

3 |	2	-10	0	3	4
|		6	-12	-36	-99
|	2	-4	-12	-33	-95
|		6	6	-18	
|	2	2	-6	-51	
|		6	24		
|	2	8	18		
|		6			
|	2	14			
|					
|	2				

$\therefore f(x) = 2(x-3)^4 + 14(x-3)^3 + 18(x-3)^2 - 51(x-3) - 95$

which is in powers of (x-3).

Replacing x by x+3, we get,

$f(x+3) = 2x^4 + 14x^3 + 18x^2 - 51x - 95$

==

Ex. Express the polynomial $4x^5 - 6x^3 + 2x^2 + 10$ in powers of $x - 2$.

Hence find f(x+2) in powers of x.

Solution: To express the polynomial $4x^5 - 6x^3 + 2x^2 + 10$

i.e. $4x^5 + 0x^4 - 6x^3 + 2x^2 + 0x + 10$ in powers of $x - 2$,

we use synthetic division as follows:

2	4	0	-6	2	0	10
		8	16	20	44	88
	4	8	10	22	44	98
		8	32	84	212	
	4	16	42	106	256	
		8	48	180		
	4	24	90	286		
		8	64			
	4	32	154			
		8				
	4	40				
	4					

$\therefore f(x) = 4(x-2)^5 + 40(x-2)^4 + 154(x-2)^3 + 286(x-2)^2 + 256(x-2) + 98$ which is in powers of (x-2).

Replacing x by x+2, we get,

$f(x+2) = 4x^5 + 40x^4 + 154x^3 + 286x^2 + 256x + 98$

==

Divisibility of Polynomials: Let f(x) and g(x) are polynomials over Q[x] with $f(x) \neq 0$. Then we say that $f(x)|g(x)$ i.e. f(x) divides g(x) if there exist polynomials q(x) over Q[x] such that $g(x) = f(x).q(x)$

Division Algorithm: Let f(x) and g(x) be any two polynomials over Q[x] with $g(x) \neq 0$. Then there exist polynomials r(x) and q(x) over Q[x] such that

$f(x) = g(x).q(x) + r(x)$, where $r(x) = 0$ or degree of r(x) < degree of g(x).

Here g(x) is called divisor, q(x) is called quotient and r(x) is called remainder.

Remark: i) f(x) | g(x) is read as f(x) divides g(x) or g(x) is multiple of f(x) or f(x) is divisor of g(x) or g(x) is divisible by f(x) or f(x) is factor of g(x).

Greatest Common Divisor (GCD): Let f(x) and g(x) be any two polynomials over Q[x] then the polynomials d(x) over Q[x] is called greatest common divisor (GCD) of f(x) and g(x) if

i) d(x)| f(x) and d(x)| g(x), ii) if c(x)| f(x) and c(x)| g(x) then c(x)|d(x).

GCD d(x) of f(x) and g(x) is denoted by d(x) = (f(x), g(x)).

===

Ex. Find G.C.D. of f(x) and g(x).

Where $f(x) = x^4 + x^3 + 4x^2 - x - 5$ and $g(x) = x^2 - 1$

Solution: Let $f(x) = x^4 + x^3 + 4x^2 - x - 5$ and $g(x) = x^2 - 1$

We divide f(x) by g(x)

$$\begin{array}{r|l} & x^2 + x + 5 \\ \hline x^2 - 1) & x^4 + x^3 + 4x^2 - x - 5 \\ & \underline{-x^4 \qquad + x^2} \\ & x^3 + 5x^2 - x - 5 \\ & \underline{-x^3 \qquad\quad + x} \\ & 5x^2 + 0x - 5 \\ & \underline{-5x^2 + 0x + 5} \\ & 0 \end{array}$$

$\therefore x^4 + x^3 + 4x^2 - x - 5 = (x^2 + x + 5)(x^2 - 1) + 0$

$\therefore$ G.C.D. of f(x) and g(x) is $x^2 - 1$.

===

Ex. Find G.C.D. of f(x) and g(x). Where $f(x) = x^2 - 1$ and $g(x) = x^3 + 7x^2 + 4x - 12$

Solution: Let $f(x) = x^2 - 1$ and $g(x) = x^3 + 7x^2 + 4x - 12$

We divide g(x) by f(x)

$$\begin{array}{r|l} & x + 7 \\ \hline x^2 - 1) & x^3 + 7x^2 + 4x - 12 \\ & \underline{-x^3 \qquad\quad + x} \\ & 7x^2 + 5x - 12 \\ & \underline{-7x^2 \qquad\quad + 7} \\ & 5x - 5 \end{array}$$

$\therefore x^3 + 7x^2 + 4x - 12 = (x+7)(x^2 - 1) + 5(x - 1)$

Again divide $x^2 - 1$ by $x - 1$

$$\begin{array}{r|l} & x + 1 \\ \hline x - 1) & x^2 + 0x - 1 \\ & \underline{-x^2 + x} \\ & x - 1 \\ & \underline{-x + 1} \\ & 0 \end{array}$$

$\therefore x^2 - 1 = (x+1)(x - 1) + 0 = (x+1)(x-1)$

$\therefore$ G.C.D. of f(x) and g(x) is (x-1).

===

Ex. Find G.C.D. of f(x) and g(x). Where $f(x) = x^4 - x^3 - 2x + 2$ and $g(x) = x^3 + x - 2$

Solution: Let $f(x) = x^4 - x^3 - 2x + 2$ and $g(x) = x^3 + x - 2$

We divide g(x) by f(x)

$$
\begin{array}{r}
x-1 \\
x^3 + x - 2 \overline{) \, x^4 - x^3 + 0x^2 - 2x + 2} \\
\underline{-x^4 \qquad - x^2 + 2x } \\
- x^3 - x^2 + 0x + 2 \\
\underline{x^3 + 0x^2 + x - 2} \\
- x^2 + x + 0
\end{array}
$$

$\therefore x^4 - x^3 + 0x^2 - 2x + 2 = (x-1)(x^3 + x - 2) + (- x^2 + x)$

Again divide $x^3 + x - 2$ by $(- x^2 + x)$

$$
\begin{array}{r}
-x-1 \\
- x^2 + x \overline{) \, x^3 + 0x^2 + x - 2} \\
\underline{-x^3 + x^2 } \\
x^2 + x - 2 \\
\underline{- x^2 + x } \\
2x - 2
\end{array}
$$

$\therefore x^3 + x - 2 = (-x-1)(- x^2 + x) + 2(x - 1)$

Again divide $- x^2 + x$ by $(x - 1)$

$$
\begin{array}{r}
-x \\
x - 1 \overline{) \, - x^2 + x} \\
\underline{x^2 - x} \\
0
\end{array}
$$

$\therefore - x^2 + x = (-x)(x - 1) + 0$

$\therefore$ G.C.D. of f(x) and g(x) is (x-1).

==

Ex. If f(x) and g(x) are polynomials over Q[x] such that $f(x) \mid g(x)$ and $g(x) \mid f(x)$, then prove that $\exists\, c \in Q$ such that $g(x) = cf(x)$.

Proof: Let f(x) and g(x) are polynomials over Q[x]

such that $f(x) \mid g(x)$ and $g(x) \mid f(x)$

$\therefore\ \exists\, q(x)$ and $r(x) \in Q[x]$ such that

$g(x) = q(x)f(x)$ and $f(x) = r(x)g(x)$ …(1)

$\therefore f(x) = r(x)q(x)f(x)$

$\therefore r(x)q(x) = 1$

$\therefore r(x)q(x) = 1$

$\therefore$ degree of q(x) = degree of r(x) = 0

$\therefore$ q(x) and r(x) both are constants say q(x) = c and r(x) = d

$\therefore g(x) = cf(x)$ by(1) with $c \in Q$. Hence proved.

==

Remainder Theorem: If a polynomial f(x) of degree $n > 1$ is divided by $(x-\alpha)$, where α is any constant, then remainder is $f(\alpha)$.

Proof: Let $f(x) = a_0x^n + a_1x^{n-1} + \ldots\ldots + a_{n-1}x + a_n$ be a polynomials of degree $n>1$.

For any constant α, put $x- \alpha = t$ i.e. $x = t + \alpha$.

$\therefore f(x) = a_0(t + \alpha)^n + a_1(t + \alpha)^{n-1} + \ldots\ldots + a_{n-1}(t + \alpha) + a_n$

$= a_0[t^n+nt^{n-1}\alpha+\ldots\ldots+nt\alpha^{n-1}+\alpha^n]$

$+ a_1[t^{n-1}+(n-1)t^{n-2}\alpha+\ldots\ldots+(n-1)t\alpha^{n-2}+\alpha^{n-1}]+ \ldots+ a_{n-1}t + a_{n-1}\alpha + a_n$

$= [a_0\alpha^n + a_1\alpha^{n-1} + \ldots\ldots + a_{n-1}\alpha + a_n]+t$[polynomial in t of degree n-1]

$= f(\alpha) + (x- \alpha)$[polynomial in x of degree n-1] $\because t = x- \alpha$

$\therefore f(x)= (x- \alpha)$[polynomial in x of degree n-1] $+ f(\alpha)$

Hence when a polynomial f(x) of degree $n > 1$ is divided by $(x-\alpha)$ the remainder is $f(\alpha)$ is proved.

==

Factor Theorem: A constant α is a root of polynomial equation $f(x) = 0$ if and only if $(x-\alpha)$ is a factor of f(x).

Proof: Let $f(x) = a_0x^n + a_1x^{n-1} + \ldots\ldots + a_{n-1}x + a_n= 0$ be a polynomial equation.

For any constant α, by remainder theorem.

$f(x)= (x- \alpha)$[polynomial in x of degree n-1] $+ f(\alpha)$

If α is a root of polynomial equation $f(x) = 0$, then $f(\alpha) = 0$.

$\therefore f(x) = (x- \alpha)$[polynomial in x of degree n-1]

Hence $(x-\alpha)$ is a factor of f(x).

Conversely: If $(x-\alpha)$ is a factor of f(x), then $f(x) = (x- \alpha)q(x)$

$\therefore f(\alpha) = (\alpha - \alpha)q(\alpha)= 0$ i.e. α satisfies an equation $f(x) = 0$.

$\therefore \alpha$ is a root of polynomial equation $f(x) = 0$ is proved.

==

Fundamental Theorem of Algebra: Every polynomial is factorized into a product of linear and irreducible quadratic factors.

Multiplicity of roots: If a root α repeated m times, then α is called root of multiplicity m.

Note: i) A polynomial equation $f(x) = 0$ of degree $n \geq 1$ has exactly n roots.

ii) If α is a root of multiplicity r of a polynomial equation $f(x) = 0$ then α is a root of multiplicity r-1 of $f'(x) = 0$.

iii) A polynomial f(x) of degree n can't vanish for more than n values of x.

iv) If $a+\sqrt{b}$ is a root of the real polynomial equation $f(x) = 0$, then $a-\sqrt{b}$ is also a root of equation $f(x) = 0$.

v) If $a+ib$ is a root of the real polynomial equation $f(x) = 0$, then $a-ib$ is also root of $f(x) = 0$.

vi) If a rational number $\frac{p}{q}$ is a root of polynomial equation $f(x) = a_0x^n + a_1x^{n-1} + \ldots\ldots + a_{n-1}x + a_n = 0$, where $a_i \in Z$ then $p \mid a_0$ and $q \mid a_n$.

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Ex. Find the rational root of the equation $15x^3 - 16x^2 - x + 2 = 0$

Solution: Let $\frac{p}{q}$ be rational root of the given equation $15x^3 - 16x^2 - x + 2 = 0$

Comparing given equation $15x^3 - 16x^2 - x + 2 = 0$ with the equation $a_0x^3 + a_1x^2 + a_2x + a_3 = 0$, we get $a_0 = 15$ and $a_3 = 2$.

By condition of rational root $p \mid a_0$ and $q \mid a_3 \Longrightarrow p \mid 15$ and $q \mid 2$

$\Longrightarrow p = \pm1, \pm3, \pm5, \pm15$ and $q = \pm1, \pm2$

$\therefore$ $\frac{p}{q} = 1$ satisfies the given equation.

$\therefore$ (x-1) is factor of given equation.

By actual division

$$\begin{array}{r l} & \underline{15x^2 - x - 2} \\ x-1) & 15x^3 - 16x^2 - x + 2 \\ & \underline{-15x^3 + 15x^2} \\ & \quad - x^2 - x + 2 \\ & \quad \underline{x^2 - x} \\ & \qquad -2x + 2 \\ & \qquad \underline{2x - 2} \\ & \qquad\quad 0 \end{array}$$

$\therefore 15x^3 - 16x^2 - x + 2 = (15x^2 - x - 2)(x - 1) + 0$

Now roots of equation $15x^2 - x - 2 = 0$ are

$x = \frac{1 \pm \sqrt{1+120}}{30} = \frac{1 \pm 11}{30} = \frac{12}{30}$ or $-\frac{10}{30}$ i.e. $\frac{2}{5}$ or $-\frac{1}{3}$

$\therefore$ The rational root of the given equation are 1, $\frac{2}{5}$ or $-\frac{1}{3}$

===

Rule to find common roots of polynomial equations:

i) The common roots of polynomial equations are the roots of their g.c.d.

ii) To find the repeated roots of polynomial equation f(x) = 0, first find g.c.d of f(x) and f '(x).

iii) If α is a root of f '(x) repeated n times, then α is a root of f(x) repeated (n+1) times.

===

Ex. Solve the equation $16x^4 - 24x^2 + 16x - 3 = 0$

Solution: Let $f(x) = 16x^4 - 24x^2 + 16x - 3 = 0$

$\therefore f'(x) = 64x^3 - 48x + 16 = 16(4x^3 - 3x + 1)$

Now we find g.c.d. of f(x) and f '(x) i.e. $16x^4 - 24x^2 + 16x - 3$ and $4x^3 - 3x + 1$

$$\begin{array}{r l} & \underline{4x - 2} \\ 4x^3-3x+1) & 16x^4 - 24x^2 + 16x - 3 \\ & \underline{-16x^4 + 12x^2 - 4x} \\ & \qquad - 12x^2 + 12x - 3 \end{array}$$

As $16x^4 - 24x^2 + 16x - 3 = (4x - 2)(4x^3 - 3x + 1) - 12x^2 + 12x - 3$

$= 2(2x - 1)(4x^3 - 3x + 1) - 3(4x^2 - 4x + 1)$

$$\begin{array}{r|l} & x+1 \\ \hline 4x^2-4x+1) & 4x^3-3x+1 \\ & \underline{-4x^3+4x^2-x} \\ & \quad 4x^2-4x+1 \\ & \quad \underline{-4x^2+4x-1} \\ & \qquad\qquad 0 \end{array}$$

$\therefore 4x^3-3x+1 = (x+1)(4x^2-4x+1)+0$

$\therefore$ g.c.d of f(x) and f '(x) is $(4x^2-4x+1)$

Now roots of equation $4x^2-4x+1=0$ i.e. $(2x-1)^2=0$ are $\frac{1}{2}, \frac{1}{2}$

which are the common roots of f(x) and f '(x).

$$\begin{array}{r|l} & 4x^2+4x-3 \\ \hline 4x^2-4x+1) & 16x^4+0x^3-24x^2+16x-3 \\ & \underline{-16x^4+16x^3-4x^2} \\ & \quad 16x^3-28x^2+16x-3 \\ & \quad \underline{-16x^3+16x^2-4x} \\ & \qquad -12x^2+12x-3 \\ & \qquad \underline{12x^2-12x+3} \\ & \qquad\qquad 0 \end{array}$$

$\therefore 16x^4+0x^3-24x^2+16x-3 = (4x^2-4x+1)(4x^2+4x-3)$

The roots of factor $4x^2+4x-3=0$ are

$x = \frac{-4\pm\sqrt{16+48}}{8} = \frac{-4\pm 8}{8} = \frac{4}{8}$ or $-\frac{12}{8}$ i.e. $\frac{1}{2}$ or $-\frac{3}{2}$

$\therefore$ The root of the given equation $16x^4-24x^2+16x-3=0$ are $\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$ and $-\frac{3}{2}$

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MULTIPLE CHOICE QUESTIONS [MCQ'S]

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1) If $f(x) = a_o+a_1x+a_2x^2+a_3x^3+ \dots\dots +a_nx^n$ with $a_n \neq 0$ is called …… of degree n.
   A) polynomial B) equation C) linear equation D) None of these

2) If $f(x) = a_o+a_1x+a_2x^2+a_3x^3+ \dots\dots +a_nx^n$ with $a_n=1$ is called …… polynomial of degree n.
   A) quadratic B) linear C) monic D)None of these

3) Polynomial $f(x) = a_o+a_1x+a_2x^2+a_3x^3+ \dots\dots +a_nx^n$ is called monic polynomial of degree n, if a_n = ……
   A) -1 B) 0 C) 1 D)None of these

4) If coefficient of highest degree term of polynomial is one then is called ……
   A) linear polynomial B) quadratic polynomial
   C) cubic polynomial D) monic polynomial

5) If f(x) = $a_0 + a_1x + a_2x^2$ +------+a_nx^n is polynomial, then its constant term is ……
   A) a_0 B) a_1 C) a_n D) None of these

6) $f(x) = x^5 + 4x^3 + 2$ is a……… polynomial of degree 5.

A) linear B) quadratic C) cubic D) monic

7) A polynomial of degree 1 is called …… polynomial.

A) linear B) quadratic C) cubic D) None of these

8) A polynomial of degree 2 is called …… polynomial.

A) linear B) quadratic C) cubic D) None of these

9) A polynomial of degree 3 is called …… polynomial.

A) linear B) quadratic C) cubic D) None of these

10) A polynomial of degree 4 is called …… polynomial.

A) linear B) biquadratic C) cubic D) None of these

11) 5x + 3 is polynomial of degree is ……

A) 0 B) 1 C) 2 D) None of these

12) $x^4 + 7x^3 + 8x^2 + 9x = 0$ is …… polynomial.

A) quadratic B) cubic C) biquadratic D) None of these

13) Two polynomials f(x) = $a_0 + a_1x + a_2x^2$ +-------+a_mx^m and g(x) = $b_0 + b_1x + b_2x^2$ +-------+b_nx^n in Q[x] are equal if ……

A) $a_i = b_i$ ∀ i and m = n B) $a_i = b_i$ ∀ i and m ≠ n

C) $a_i \neq b_i$ D) None of these

14) If $f(x) = a_o+a_1x+a_2x^2+a_3x^3+$ ……. $+a_nx^n = 0$ with $a_n \neq 0$ is called … of degree n.

A) polynomial B) equation C) rational equation D) None of these

15) If $f(x) = a_o+a_1x+a_2x^2+a_3x^3+$ ……. $+a_nx^n = 0$ with $a_n = 1$ is called …… polynomial equation of degree n.

A) quadratic B) linear C) monic D)None of these

16) $f(x) = a_o+a_1x+a_2x^2+a_3x^3+$ ……. $+a_nx^n = 0$ is called monic polynomial equation of degree n, if a_n = ……

A) -1 B) 0 C) 1 D)None of these

17) An equation of degree one is called …… equation.

A) linear B) quadratic C) cubic D) None of these

18) An equation of degree two is called …… equation.

A) linear B) quadratic C) cubic D) None of these

19) An equation of degree three is called …… equation.

A) linear B) quadratic C) cubic D) None of these

20) An equation of degree four is called …… equation.

A) linear B) quadratic C) biquadratic D) None of these

21) $f(x) = 2 - 2x + 9x^2 - x^3$ is polynomial of degree ……

A) 3 B) 2 C) 1 D) None of these

22) $g(x) = 1 - 5x^2 + \frac{7}{2}x^4$ is polynomial of degree ……

A) 2 B) 4 C) 5 D) None of these

23) $x^5+6x^3-5x^2+7=0$ is a polynomial equation of degree ……

A) 2 B) 4 C) 5 D) None of these

24) If f(x) is polynomial of degree m and g(x) is polynomial of degree n then degree of [f(x)+g(x)] is ……

A) max.{m, n} B) min.{m, n} C) m+n D) None of these

25) If f(x) = $a_0 + a_1x + a_2x^2 + \ldots + a_mx^m$ and g(x) = $b_0 + b_1x + b_2x^2 + \ldots + b_nx^n$ then deg[f(x)+g(x)] = ……

A) max.{m, n} B) m+n C) mn D) min.{m, n}

26) If f(x) is polynomial of degree 4 and g(x) is polynomial of degree 5 then degree of [f(x)+g(x)] is ……

A) 9 B) 5 C) 4 D) None of these

27) If f(x) = $3 + 5x - 7x^2 + \frac{1}{2}x^3$ and g(x) = $5 + 7x - 5x^2 + 2x^3 + 9x^4$, then degree of [f(x)+g(x)] is ……

A) 9 B) 5 C) 4 D) None of these

28) If f(x) = $1 - 3x + 2x^2$ and g(x) = $3 - \frac{1}{2}x + 4x^2 - x^3$, then f(x) + g(x) = ……

A) $3 - \frac{7}{2}x + 6x^2 - x^3$ B) $4 - \frac{7}{2}x + 6x^2 - x^3$

C) $3 - \frac{1}{2}x + 6x^2 - x^3$ D) None of these

29) If f(x) = $3 + 5x - 7x^2 + \frac{1}{2}x^3$ and g(x) = $5 + 7x - 5x^2 + 2x^3 + 9x^4$, then f(x) + g(x) = ……

A) $8 + 12x - 12x^2 + \frac{5}{2}x^3 + 9x^4$ B) $8 + 12x - 12x^2 + \frac{1}{2}x^3 + 9x^4$

C) $8 + 12x - 12x^2 + 5x^3 + 9x^4$ D) None of these

30) If f(x) is polynomial of degree m and g(x) is polynomial of degree n then degree of [f(x).g(x)] is ……

A) max.{m, n} B) min.{m, n} C) m+n D) None of these

31) If f(x) = $a_0 + a_1x + a_2x^2 + \ldots + a_mx^m$ and g(x) = $b_0 + b_1x + b_2x^2 + \ldots + b_nx^n$ then deg[f(x).g(x)] = ……

A) max.{m, n} B) m+n C) mn D) min.{m, n}

32) If f(x) is polynomial of degree 4 and g(x) is polynomial of degree 5 then degree of [f(x).g(x)] is ……

A) 9 B) 5 C) 4 D) None of these

33) If f(x) is divided by (x-α) where α is constant, then remainder is….

A) α B) f(α) C) 0 D) None of these

34) If a constant number α is root of an equation f(x) = 0, then (x-α) is…of f(x).

A) root B) factor C) remainder D) None of these

35) If a constant number α is root of an equation f(x) = 0, then ……

A) (x+α)|f(x) B) f(x)|(x-α) C) (x-α)|f(x) D) None of these

36) If f(x) = $2x^3 + 3x + 2$ is divided by (x-3), then by Horner's method of synthetic division remainder is….

A) 65 B) 57 C) 18 D) 2

37) If $f(x) = x^5 + 3x^4 + 5x^2 - 2$ is divided by (x-3), then by Horner's method of synthetic division remainder is….

A) 3 B) -2 C) 529 D) 286

38) The remainder when $4x^4 - 2x^3 + 3x - 3$ is divided by x + 1 is ……

A) 0 B) -1 C) 1 D) None of these

39) A constant α is root of polynomial equation f(x) = 0, if …… is factor of f(x).

A) x-α B) f(α) C) x+ α D) None of these

40) A polynomial equation f(x) = 0 of degree n ≥ 1 has exactly …… roots.

A) 1 B) n-1 C) n D) None of these

41) A cubic polynomial equation f(x) = 0 has exactly …… roots.

A) 3 B) 2 C) 1 D) None of these

42) A cubic polynomial equation f(x) = 0 has minimum …… real roots.

A) 3 B) 2 C) 1 D) None of these

43) If $a + \sqrt{b}$ is a real root of polynomial equation f(x) = 0 then …… is also a root of f(x) = 0.

A) a B) $\sqrt{b}$ C) $a - \sqrt{b}$ D) None of these

44) If $\alpha + i\beta$ is a root of polynomial equation f(x) = 0 then …… is also a root of f(x) = 0.

A) $\alpha - i\beta$ B) α C) β D) None of these

45) If f(x) and g(x) are polynomials over Q[x] such that g(x) ≠ 0 then there exists polynomials q(x) and r(x) over Q[x] such that f(x) = q(x).g(x) + r(x) where r(x) = 0 or deg of r(x) < deg of g(x) then r(x) is called ……

A) divisor B) quotient C) remainder D) None of these

46) If f(x) and g(x) are polynomials over Q[x] such that g(x) ≠ 0 then there exists polynomials q(x) and r(x) over Q[x] such that f(x) = q(x).g(x) + r(x) where r(x) = 0 or deg of r(x) < deg of g(x) then g(x) is called ……

A) divisor B) quotient C) remainder D) None of these

47) If f(x) and g(x) are polynomials over Q[x] such that g(x) ≠ 0 then there exists polynomials q(x) and r(x) over Q[x] such that f(x) = q(x).g(x) + r(x) where r(x) = 0 or deg of r(x) < deg of g(x) then q(x) is called ……

A) divisor B) quotient C) remainder D) None of these

48) d(x) is g.c.d of f(x) and g(x), then ……

A) d(x)|f(x) and d(x)|g(x) B) f(x)|d(x) and g(x)|d(x)

C) d(x)|f(x) and f(x)|g(x) D) None of these

49) g.c.d of f(x) and g(x) is denoted by

A) (f(x), g(x)) B) [f(x), g(x)] C) {f(x), g(x)} D) None of these

50) The common roots of two given polynomials are the roots of their …..

A) g.c.d. B) quotient C) remainder D) None of these

51) To find the repeated roots of given polynomial equation f(x) = 0. We first find the g.c.d. of …..

A) f(x) = 0 & f ''(x) = 0 B) f(x) = 0 & f '(x) = 0

C) f '(x) = 0 & f ''(x) = 0 D) None of these

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UNIT-3. THEORY OF EQUATIONS-I

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Polynomials: If $a_0, a_1, a_2, \ldots\ldots, a_n$ are real numbers with $a_0 \neq 0$, then
$P_n(x)$ or $f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots\ldots + a_{n-1}x + a_n$ is called a general polynomial of one variable x of degree n.

Polynomials Equation: If $a_0, a_1, a_2, \ldots\ldots, a_n$ are real numbers with $a_0 \neq 0$ and x is variable, then $P_n(x)$ or $f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots\ldots + a_{n-1}x + a_n = 0$ is called a polynomial equation of degree n

Root of an Equation: The value of x which satisfies the given equation $f(x) = 0$ is called root of an equation.

Note: The values of roots may be real or complex.

Solution of an Equation: The set of all roots an equation $f(x) = 0$ is called it's solution.

Linear Equation: A polynomial equation $ax + b = 0$ of degree 1 is called linear equation.

Quadratic Equation: A polynomial equation $ax^2 + bx + c = 0$ of degree 2 is called quadratic equation.

Cubic Equation: A polynomial equation $ax^3 + bx^2 + cx + d = 0$ of degree 3 is called cubic equation.

Biquadratic Equation: A polynomial equation $ax^4 + bx^3 + cx^2 + dx + e = 0$ of degree 4 is called biquadratic equation.

Quintic Equation: A polynomial equation $ax^5 + bx^4 + cx^3 + dx^2 + ex + f = 0$ of degree 5 is called quintic equation.

Sextic Equation: A polynomial equation $ax^6 + bx^5 + cx^4 + dx^3 + ex^2 + fx + g = 0$ of degree 6 is called sextic equation.

e.g. i) $3x^2 + 4x - 7 = 0$ is a quadratic equation.
ii) $4x^3 - 7x + 1 = 0$ is a cubic equation.
iii) $x^4 - 8x^3 + 5x - 7 = 0$ is a biquadratic equation.

Notation: $\alpha_1, \alpha_2, \alpha_3, \ldots\ldots, \alpha_n$ are n numbers in sequence, then

$\sum \alpha_1 = \alpha_1 + \alpha_2 + \alpha_3 + \ldots\ldots + \alpha_n$

$\sum \alpha_1\alpha_2$ = The sum of all possible products of α_i's taken two distinct α_i at a time.

$\sum \alpha_1\alpha_2\alpha_3$ = The sum of all possible products of α_i's taken three distinct α_i at a time.

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Relation between roots and coefficient of general polynomial equation:

Let $f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots\ldots + a_{n-1}x + a_n = 0$ with $a_0 \neq 0$ be a general polynomial equation of one variable x of degree n and $\alpha_1, \alpha_2, \alpha_3, \ldots\ldots, \alpha_n$ are n roots of $f(x) = 0$, then

$f(x) = a_0(x-\alpha_1)(x-\alpha_2)(x-\alpha_3)\ldots\ldots(x-\alpha_n)$

$\therefore a_0x^n + a_1x^{n-1} + a_2x^{n-2}+\ldots\ldots+a_{n-1}x + a_n = a_0(x-\alpha_1)(x-\alpha_2)(x-\alpha_3)\ldots\ldots(x-\alpha_n)$

$= a_0[x^n-(\alpha_1+\alpha_2+\alpha_3+\ldots\ldots+\alpha_n)x^{n-1} + (\alpha_1\alpha_2+\alpha_1\alpha_3+\ldots\ldots+\alpha_{n-1}\alpha_n)x^{n-2}$

$-(\alpha_1\alpha_2\alpha_3+\alpha_1\alpha_2\alpha_3+\ldots+\alpha_{n-2}\alpha_{n-1}\alpha_n)x^{n-3} + \ldots + (-1)^n(\alpha_1\alpha_2\alpha_3\ldots\alpha_{n-1}\alpha_n)]$

Comparing the coefficient of x^{n-1}, x^{n-2}, …, x and constant terms, we get,

$-a_0(\alpha_1+\alpha_2+\alpha_3+\ldots\ldots+\alpha_n) = a_1$ i.e. $\sum\alpha_1 = -\frac{a_1}{a_0}$

$a_0(\alpha_1\alpha_2+\alpha_1\alpha_3+\ldots\ldots+\alpha_{n-1}\alpha_n) = a_2$ i.e. $\sum\alpha_1\alpha_2 = \frac{a_2}{a_0}$

$-a_0(\alpha_1\alpha_2\alpha_3+\alpha_1\alpha_2\alpha_3+\ldots+\alpha_{n-2}\alpha_{n-1}\alpha_n) = a_3$ i.e. $\sum\alpha_1\alpha_2\alpha_3 = -\frac{a_3}{a_0}$

…

…

…

$a_0(-1)^n(\alpha_1\alpha_2\alpha_3\ldots\alpha_{n-1}\alpha_n) = a_n$ i.e. $\alpha_1\alpha_2\alpha_3\ldots\alpha_{n-1}\alpha_n = (-1)^n\frac{a_n}{a_0}$

==

Relation between roots and coefficient of some polynomial equations:

i) Let α and β are the roots of a quadratic equation $ax^2 + bx + c = 0$, then relation between roots and coefficients are $\sum\alpha = \alpha+\beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

ii) Let α and β are the roots of a quadratic equation $x^2 + px + q = 0$, then relation between roots and coefficients are $\sum\alpha = \alpha+\beta = -p$ and $\alpha\beta = q$

iii) Let α, β and γ are the roots of a cubic equation $ax^3 + bx^2 + cx + d = 0$, then relation between roots and coefficients are

$\sum\alpha = \alpha+\beta+\gamma = -\frac{b}{a}$, $\sum\alpha\beta = \frac{c}{a}$ and $\alpha\beta\gamma = -\frac{d}{a}$

iv) Let α, β and γ are the roots of a cubic equation $x^3 + px^2 + qx + r = 0$, then relation between roots and coefficients are

$\sum\alpha = \alpha+\beta+\gamma = -p$, $\sum\alpha\beta = q$ and $\alpha\beta\gamma = -r$

v) Let α, β and γ are the roots of a cubic equation $x^3 - px^2 + qx - r = 0$, then relation between roots and coefficients are

$\sum\alpha = \alpha+\beta+\gamma = p$, $\sum\alpha\beta = q$ and $\sum\alpha\beta\gamma = r$

vi) Let α, β, γ and δ are the roots of a biquadratic equation $ax^4+bx^3+cx^2+dx+e = 0$, then relation between roots and coefficient are

$\sum\alpha = \alpha+\beta+\gamma+\delta = -\frac{b}{a}$, $\sum\alpha\beta = \frac{c}{a}$, $\sum\alpha\beta\gamma = -\frac{d}{a}$ and $\alpha\beta\gamma\delta = \frac{e}{a}$

==

vi) Let α, β, γ and δ are the roots of a biquadratic equation $x^4+px^3+qx^2+rx+s = 0$, then relation between roots and coefficient are

$\sum\alpha = \alpha+\beta+\gamma+\delta = -p$, $\sum\alpha\beta = q$, $\sum\alpha\beta\gamma = -r$ and $\sum\alpha\beta\gamma\delta = s$

vii) Let α, β, γ and δ are the roots of a biquadratic equation $x^4-px^3+qx^2-rx+s = 0$, then relation between roots and coefficient are

$\sum\alpha = \alpha+\beta+\gamma+\delta = p$, $\sum\alpha\beta = q$, $\sum\alpha\beta\gamma = r$ and $\sum\alpha\beta\gamma\delta = s$

Remark: i) Roots $\alpha-\beta, \alpha, \alpha+\beta$ of cubic equation are in arithmetic progression (A.P.), ii) Roots $\frac{\alpha}{\beta}, \alpha, \alpha\beta$ of cubic equation are in geometric progression (G.P.)

==

Ex.: Solve the equation $x^3 - 3x^2 - 16x + 48 = 0$, if sum of two of its roots is zero.

Solution: Let α, β and γ are the roots of the given equation $x^3 - 3x^2 - 16x + 48 = 0$, with sum of two of its roots is zero say $\beta + \gamma = 0$ ……. (1)

By relation between roots and coefficients, we have

$\sum \alpha = \alpha+\beta+\gamma = -\frac{b}{a}$ i.e. $\alpha + 0 = -\frac{(-3)}{1}$ i.e. $\alpha = 3$ by (1)

$\sum \alpha\beta = \alpha\beta+\alpha\gamma+\beta\gamma = \frac{c}{a}$ i.e. $\alpha(0) + \beta(-\beta) = \frac{-16}{1}$ by (1)

$\therefore -\beta^2 = -16$

$\therefore \beta^2 = 16$

By taking positive square root, we get,

$\therefore \beta = 4$ and $\gamma = -4$ by (1)

∴ 3, 4 and -4 are the roots of given equation.

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Ex.: Solve the equation $x^3 - 5x^2 - 16x + 80 = 0$, if sum of two of its roots is zero.

Solution: Let α, β and γ are the roots of the given equation $x^3 - 5x^2 - 16x + 80 = 0$, with sum of two of its roots is zero say $\beta + \gamma = 0$ ……. (1)

By relation between roots and coefficients, we have

$\sum \alpha = \alpha+\beta+\gamma = -\frac{b}{a}$ i.e. $\alpha + 0 = -\frac{(-5)}{1}$ i.e. $\alpha = 5$ by (1)

$\sum \alpha\beta = \alpha\beta+\alpha\gamma+\beta\gamma = \frac{c}{a}$ i.e. $\alpha(0)+\beta(-\beta) = \frac{-16}{1}$ by (1)

i.e. $\beta^2 = 16$

By taking positive square root, we get,

$\therefore \beta = 4$ and $\gamma = -4$ by (1)

∴ 5, 4 and - 4 are the roots of given equation.

==

Ex.: Solve the equation $x^3 - 3x^2 + 4 = 0$, if two its roots are equal.

Solution: Let α, β and γ are the roots of the given equation

$x^3 - 3x^2 + 4 = 0$ with two roots are equal say $\gamma = \beta$ …… (1)

By relation between roots and coefficients, we have

$\sum \alpha = \alpha + \beta + \gamma = -\frac{b}{a}$ i.e. $\alpha + 2\beta = -\frac{(-3)}{1}$ by (1)

i.e. $\alpha + 2\beta = 3$ …… (2)

$\sum \alpha\beta = \alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$ i.e. $2\alpha\beta + \beta^2 = 0$ by (1)

i.e. $\beta(2\alpha + \beta) = 0$

$\therefore 2\alpha + \beta = 0$ …… (3) $\because \beta = 0$ does not satisfy given equation.

Consider 2(2) – (3)

$2\alpha + 4\beta - 2\alpha - \beta = 6 - 0$

$\therefore 3\beta = 6 \Rightarrow \beta = 2$

From (2), we get,

$\alpha + 4 = 3 \Rightarrow \alpha = -1$

∴ -1, 2 and 2 are the roots of given equation.

==

Ex.: Solve the equation $x^3 - 7x^2 + 36 = 0$ whose one of the root is double the other.

Solution: Let α, β and γ are the roots of the given equation $x^3 - 7x^2 + 36 = 0$, whose one of the root is double the other say $\gamma = 2\beta$ ……. (1)

By relation between roots and coefficients, we have

$\sum \alpha = \alpha+\beta+\gamma = -\frac{b}{a}$ i.e. $\alpha+\beta+2\beta = -\frac{(-7)}{1}$ by (1)

i.e. $\alpha + 3\beta = 7$ …… (2)

$\sum \alpha\beta = \alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$ i.e. $\alpha\beta + \alpha(2\beta) + \beta(2\beta) = \frac{0}{1}$ by (1)

i.e. $3\alpha\beta + 2\beta^2 = 0$

i.e. $\beta(3\alpha + 2\beta) = 0$

$\therefore 3\alpha + 2\beta = 0$ …… (3) $\because \beta = 0$ does not satisfy given equation.

Consider 3(2) – (3)

$3\alpha + 9\beta - 3\alpha - 2\beta = 21 - 0$

i.e. $7\beta = 21$

$\therefore \beta = 3$ and $\gamma = 2(3) = 6$ by (1)

From (2), we get,

$\alpha + 3(3) = 7$

$\therefore \alpha = -2$

∴ -2, 3 and 6 are the roots of given equation.

===

Ex.: Solve the equation $x^3 - 9x^2 + 23x - 15 = 0$ whose roots are in A. P.

Solution: Let α - β, α and α + β are the roots of the given equation $x^3 - 9x^2 + 23x - 15 = 0$ in A. P.

By relation between roots and coefficients, we have

$\alpha - \beta + \alpha + \alpha + \beta = -\frac{b}{a}$ i.e. $3\alpha = -\frac{(-9)}{1}$

i.e. $3\alpha = 9 \Rightarrow \alpha = 3$

$(\alpha - \beta)\alpha(\alpha + \beta) = -\frac{d}{a}$ i.e. $\alpha(\alpha^2 - \beta^2) = -\frac{(-15)}{1}$

i.e. $3(9 - \beta^2) = 15$

$\therefore 9 - \beta^2 = 5$

$\therefore \beta^2 = 4$

By taking positive square root, we get,

$\therefore \beta = 2$

∴ 3-2, 3 and 3+2 i.e. 1, 3 and 5 are the roots of given equation.

===

Ex.: Solve the equation $x^3 - 3x^2 - 6x + 8 = 0$ if the roots are in arithmetic progression (A. P.).

Solution: Let α - β, α and α + β are the roots of the given equation $x^3 - 3x^2 - 6x + 8 = 0$ in A. P.

By relation between roots and coefficients, we have

$\alpha - \beta + \alpha + \alpha + \beta = -\frac{b}{a}$ i.e. $3\alpha = -\frac{(-3)}{1}$ by(1)

i.e. $3\alpha = 3 \Rightarrow \alpha = 1$

$(\alpha - \beta)\alpha(\alpha + \beta) = -\frac{d}{a}$ i.e. $\alpha(\alpha^2-\beta^2) = -\frac{(8)}{1}$

i.e. $1 - \beta^2 = -8$

$\therefore \beta^2 = 9$

By taking positive square root, we get,

$\therefore \beta = 3$

$\therefore$ 1-3, 1 and 1+3 i.e. -2, 1 and 4 are the roots of given equation.

==

Ex.: Find the condition that the roots of $x^3 - px^2 + qx - r = 0$ are in A. P.

Solution: Let $\alpha - \beta$, α and $\alpha + \beta$ are the roots of the given equation

$x^3 - px^2 + qx - r = 0$ in A. P.

By relation between roots and coefficients, we have

$\alpha - \beta + \alpha + \alpha + \beta = -\frac{b}{a}$ i.e. $3\alpha = -\frac{(-p)}{1}$ by(1)

i.e. $3\alpha = p \Rightarrow \alpha = \frac{p}{3}$

As $\alpha = \frac{p}{3}$ is root of given equation.

$\therefore (\frac{p}{3})^3 - p(\frac{p}{3})^2 + q(\frac{p}{3}) - r = 0$

$\therefore \frac{p^3}{27} - \frac{p^3}{9} + \frac{pq}{3} - r = 0$

$\therefore p^3 - 3p^3 + 9pq - 27r = 0$

$\therefore -2p^3 + 9pq - 27r = 0$

$\therefore 2p^3 - 9pq + 27r = 0$ be the required condition.

==

Ex.: Find the condition that $x^3 + px^2 + qx + r = 0$ should have the roots α, β related by $\alpha\beta + 1 = 0$.

Solution: Let α, β and γ are the roots of the given equation

$x^3 + px^2 + qx + r = 0$ with $\alpha\beta + 1 = 0$ i.e. $\alpha\beta = -1$ …… (1)

By relation between roots and coefficients, we have

$\alpha\beta\gamma = -\frac{d}{a}$ i.e. $(-1)\gamma = -\frac{r}{1}$ by (1)

i.e. $\gamma = r$

As $\gamma = r$ is root of given equation.

$\therefore r^3 + pr^2 + qr + r = 0$

i.e. $r^2 + pr + q + 1 = 0$ be the required condition.

==

Symmetric Functions of Roots: An expression in roots which is remain same after interchange of roots is called symmetric functions of roots.

e.g: i) $\alpha+\beta$, $\alpha\beta$, $\alpha^2+\beta^2$, $\frac{1}{\alpha} + \frac{1}{\beta}$, $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ etc. are symmetric functions of two roots α & β.

ii) $\alpha+\beta+\gamma$, $\alpha\beta+\beta\gamma+\gamma\alpha$, $\alpha\beta\gamma$, $\alpha^2+\beta^2+\gamma^2$, $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$, $\frac{\alpha}{\beta}+\frac{\beta}{\gamma}+\frac{\gamma}{\alpha}$ etc. are the symmetric functions of two roots α, β & γ.

iii) $\alpha-\beta$, $\frac{\alpha}{\beta}$, $\alpha^2-\beta^2$, $\frac{1}{\alpha}-\frac{1}{\beta}$, $\frac{\alpha}{\beta}-\frac{\beta}{\alpha}$ are not symmetric functions.

==

Ex.: If α and β are the roots of $ax^2 + bx + c = 0$, then find the values of

i) $\alpha^2+\beta^2$ ii) $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$ iii) $\alpha^3+\beta^3$ iv) $\frac{\alpha^2}{\beta^2}+\frac{\beta^2}{\alpha^2}$

Solution: Let α and β are the roots of the equation $ax^2 + bx + c = 0$

$\therefore \alpha+\beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$ …… (1)

i) $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = (-\frac{b}{a})^2 - 2(\frac{c}{a})$ by (1)

$= \frac{b^2-2ac}{a^2}$ …… (2)

ii) $\frac{\alpha}{\beta}+\frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} = (\frac{a}{c})(\frac{b^2-2ac}{a^2})$ by (1) and (2)

$= \frac{b^2-2ac}{ac}$

iii) $\alpha^3+\beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta) = (-\frac{b}{a})^3 - 3(\frac{c}{a})(-\frac{b}{a})$ by(1)

$= \frac{-b^3}{a^3} + \frac{3bc}{a^2}$

$= \frac{3abc-b^3}{a^3}$

iv) $\frac{\alpha^2}{\beta^2}+\frac{\beta^2}{\alpha^2} = \frac{\alpha^4+\beta^4}{\alpha^2\beta^2} = \frac{(\alpha^2+\beta^2)^2-2(\alpha\beta)^2}{(\alpha\beta)^2} = \frac{(\frac{b^2-2ac}{a^2})^2-2(\frac{c}{a})^2}{(\frac{c}{a})^2}$ by (1) and (2)

$= \frac{(b^2-2ac)^2-2(ac)^2}{(ac)^2}$

$= \frac{b^4-4ab^2c+2a^2c^2}{a^2c^2}$

==

Ex.: If α and β are the roots of $x^2 - 5x + 1 = 0$, then find the values of

i) $\frac{1}{\alpha}+\frac{1}{\beta}$ ii) $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$ iii) $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}$ iv) $\alpha^4+\beta^4$

Solution: Let α and β are the roots of the equation $x^2 - 5x + 1 = 0$

$\therefore \alpha+\beta = -\frac{(-5)}{1} = 5$ and $\alpha\beta = \frac{1}{1} = 1$…… (1)

i) $\frac{1}{\alpha}+\frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{5}{1} = 5$ by (1)

ii) $\frac{\alpha}{\beta}+\frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta} = \frac{(5)^2-2(1)}{(1)} = 23$ by (1)

iii) $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha} = \frac{\alpha^3+\beta^3}{\alpha\beta} = \frac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{\alpha\beta} = \frac{(5)^3-3(1)(5)}{(1)}$ by(1)

$= 125 - 15$

$= 110$

iv) $\alpha^4+\beta^4 = (\alpha^2+\beta^2)^2 - 2(\alpha\beta)^2 = (23)^2 - 2(1)^2$ by (1) and (2)

$= 529 - 2$

$= 527$

==

Ex.: If α and β are the roots of $3x^2 - 4x + 7 = 0$, then find the values of

i) $\frac{1}{\alpha} + \frac{1}{\beta}$ iv) $\alpha^2+\beta^2$ ii) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ iii) $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

Solution: Let α and β are the roots of the equation $3x^2 - 4x + 7 = 0$

$\therefore \alpha+\beta = -\frac{(-4)}{3} = \frac{4}{3}$ and $\alpha\beta = \frac{7}{3}$ …… (1)

i) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{\frac{4}{3}}{\frac{7}{3}} = \frac{4}{7}$ by (1)

ii) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = (\frac{4}{3})^2 - 2(\frac{7}{3})$ by (1)

$= \frac{16}{9} - \frac{14}{3}$

$= \frac{-26}{9}$ …… (2)

iii) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{\frac{-26}{9}}{\frac{7}{3}} = \frac{-26}{9} \times \frac{3}{7} = \frac{-26}{21}$ by (1) and (2)

==

Ex.: If α, β and γ are the roots of $x^3 + px^2 + qx + r = 0$, then find the values of

i) $\sum\alpha^2$ ii) $\sum\alpha^2\beta$ iii) $\sum\alpha^3$

Solution: Let α, β and γ are the roots of $x^3 + px^2 + qx + r = 0$

$\therefore \sum\alpha = \alpha + \beta + \gamma = -\frac{p}{1} = -p,\ \sum\alpha\beta = \alpha\beta + \beta\gamma + \gamma\alpha = \frac{q}{1} = q$ and

$\alpha\beta\gamma = -\frac{r}{1} = -r$ …… (1)

i) $\sum\alpha^2 = \alpha^2 + \beta^2 + \gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\beta\gamma+\gamma\alpha)$

$= (\sum\alpha)^2 - 2\sum\alpha\beta$

$= (-p)^2 - 2(q)$

$= p^2 - 2q$ …… (2)

ii) $\sum\alpha^2\beta = \alpha^2\beta + \alpha^2\gamma + \beta^2\alpha + \beta^2\gamma + \gamma^2\alpha + \gamma^2\beta$

$= (\alpha+\beta+\gamma)(\alpha\beta+\beta\gamma+\gamma\alpha) - 3\alpha\beta\gamma$

$= (-p)(q) - 3(-r)$

$= 3r - pq$ …… (3)

iii) $\sum\alpha^3 = \alpha^3 + \beta^3 + \gamma^3 = (\alpha+\beta+\gamma)(\alpha^2+\beta^2+\gamma^2) -$

$-(\alpha^2\beta + \alpha^2\gamma + \beta^2\alpha + \beta^2\gamma + \gamma^2\alpha + \gamma^2\beta)$

$= (\sum\alpha)(\sum\alpha^2) - \sum\alpha^2\beta$

$= (-p)(p^2 - 2q) - (3r - pq)$

$= -p^3 + 2pq - 3r + pq$

$= 3pq - p^3 - 3r$

==

Ex.: If α, β and γ are the roots of $x^3 - 5x^2 - 2x + 24 = 0$, then find the values of $\sum\alpha^2$ and $\sum\alpha^2\beta$

Solution: Let α, β and γ are the roots of $x^3 - 5x^2 - 2x + 24 = 0$

$\therefore \sum\alpha = \alpha + \beta + \gamma = -\frac{(-5)}{1} = 5,\ \sum\alpha\beta = \alpha\beta + \beta\gamma + \gamma\alpha = \frac{-2}{1} = -2$ and

$\alpha\beta\gamma = -\frac{(24)}{1} = -24$ …… (1)

i) $\sum \alpha^2 = \alpha^2 + \beta^2 + \gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\beta\gamma+\gamma\alpha)$

$= (\sum \alpha)^2 - 2\sum \alpha\beta$

$= (5)^2 - 2(-2)$

$= 29$ …… (2)

ii) $\sum \alpha^2\beta = \alpha^2\beta + \alpha^2\gamma + \beta^2\alpha + \beta^2\gamma + \gamma^2\alpha + \gamma^2\beta$

$= (\alpha+\beta+\gamma)(\alpha\beta+\beta\gamma+\gamma\alpha) - 3\alpha\beta\gamma$

$= (5)(-2) - 3(-24)$

$= 62$

==

Ex.: If α, β and γ are the roots of $x^3 - 3x^2 + 4x - 1 = 0$, then find the values of

i) $\sum \alpha^2$ ii) $\sum \alpha^2\beta$ iii) $(\alpha+\beta)(\beta+\gamma)(\gamma+\alpha)$ iv) $\frac{1}{\alpha^2\beta^2} + \frac{1}{\beta^2\gamma^2} + \frac{1}{\gamma^2\alpha^2}$

Solution: Let α, β and γ are the roots of $x^3 - 3x^2 + 4x - 1 = 0$

$\therefore \sum \alpha = \alpha+\beta+\gamma = -\frac{(-3)}{1} = 3$, $\sum \alpha\beta = \alpha\beta+\beta\gamma+\gamma\alpha = \frac{4}{1} = 4$ and

$\alpha\beta\gamma = -\frac{(-1)}{1} = 1$ …… (1)

i) $\sum \alpha^2 = \alpha^2 + \beta^2 + \gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\beta\gamma+\gamma\alpha)$

$= (\sum \alpha)^2 - 2\sum \alpha\beta$

$= (3)^2 - 2(4)$ by (1)

$= 1$ …… (2)

ii) $\sum \alpha^2\beta = \alpha^2\beta + \alpha^2\gamma + \beta^2\alpha + \beta^2\gamma + \gamma^2\alpha + \gamma^2\beta$

$= (\alpha+\beta+\gamma)(\alpha\beta+\beta\gamma+\gamma\alpha) - 3\alpha\beta\gamma$

$= (3)(4) - 3(1)$ by (1)

$= 9$ …… (3)

iii) $(\alpha+\beta)(\beta+\gamma)(\gamma+\alpha) = (\alpha\beta + \alpha\gamma + \beta^2 + \beta\gamma)(\gamma+\alpha)$

$= \alpha\beta\gamma + \alpha^2\beta + \gamma^2\alpha + \alpha^2\gamma + \beta^2\gamma + \beta^2\alpha + \gamma^2\beta + \alpha\beta\gamma$

$= 2\alpha\beta\gamma + \sum \alpha^2\beta$

$= 2(1) + 9$ by (1) and (3)

$= 11$

iv) $\frac{1}{\alpha^2\beta^2} + \frac{1}{\beta^2\gamma^2} + \frac{1}{\gamma^2\alpha^2} = \frac{\gamma^2+\alpha^2+\beta^2}{\alpha^2\beta^2\gamma^2}$

$= \frac{\sum \alpha^2}{(\alpha\beta\gamma)^2}$

$= \frac{1}{(1)^2}$ by (1) and (2)

$= 1$

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MULTIPLE CHOICE QUESTIONS [MCQ'S]

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1) If $a_0, a_1, a_2, \ldots, a_n$ are real numbers with $a_0 \neq 0$, then

$f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n$ is called a ……. of one variable x of degree n

A) general equation B) general polynomial

C) linear equation D) None of these

2) If $a_0, a_1, a_2, \ldots, a_n$ are real numbers with $a_n \neq 0$, then
$f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n$ is a general polynomial of degree ……

A) 0 B) 1 C) n D) None of these

3) Constant term of a given polynomial $f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n$ is ……

A) a_0 B) a_1 C) a_n D) None of these

4) Coefficient of n^{th} degree term of a given polynomial
$f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n$ is ……

A) a_0 B) a_1 C) a_n D) None of these

5) If $a_0, a_1, a_2, \ldots, a_n$ are real numbers with $a_0 \neq 0$, then
$f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n = 0$ is called a ……. of degree n.

A) general equation B) general polynomial

C) linear equation D) None of these

6) If $a_0, a_1, a_2, \ldots, a_n$ are real numbers with $a_0 \neq 0$, then
$f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n = 0$ is a general equation of degree ……

A) 0 B) 1 C) n D) None of these

7) Constant term of a given equation $f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n$ is ……

A) a_0 B) a_1 C) a_n D) None of these

8) Coefficient of n^{th} degree term of a given equation
$f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n$ is ……

A) a_0 B) a_1 C) a_n D) None of these

9) The value of x which satisfies the given equation $f(x) = 0$ is called …… of an equation

A) root B) solution C) pole D) None of these

10) The values of roots of equation $f(x) = 0$ are……

A) only real B) only complex

C) may be real or complex D) None of these

11) The set of all roots of an equation $f(x) = 0$ is called it's …… set.

A) solution B) open C) closed D) None of these

12) A polynomial equation $ax + b = 0$ of degree 1 is called …… equation.

A) cubic B) quadratic C) linear D) None of these

13) Degree of linear equation is ……

A) 1 B) 2 C) 3 D) 4

14) A polynomial equation $ax^2 + bx + c = 0$ of degree 2 is called …… equation.

A) cubic B) quadratic C) linear D) None of these

15) Degree of quadratic equation is ……

A) 1 B) 2 C) 3 D) 4

16) A polynomial equation $ax^3 + bx^2 + cx + d = 0$ of degree 3 is called …… equation.

A) cubic B) quadratic C) linear D) None of these

17) Degree of cubic equation is ……

A) 1 B) 2 C) 3 D) 4

18) A polynomial equation $ax^4 + bx^3 + cx^2 + dx + e = 0$ of degree 4 is called …… equation.

A) cubic B) biquadratic C) linear D) None of these

19) Degree of a biquadratic equation is ……

A) 5 B) 4 C) 3 D) 2

20) Degree of an equation $x^4 + x^3 + x^2 + x + 1 = 0$ is ……

A) 1 B) 4 C) 3 D) 2

21) Degree of an equation $x^5 - x^4 + x^3 - x^2 + x - 1 = 0$ is ……

A) 5 B) 4 C) 1 D) -1

22) Degree of an equation $ax^5 + bx^4 + cx^3 + dx^2 + ex + f = 0$ is ……

A) 5 B) 4 C) 3 D) 2

23) Degree of an equation $x^6 + 4x^4 - 3x + 9 = 0$ is ……

A) 5 B) 4 C) 6 D) 9

24) If α and β are the roots of a quadratic equation $ax^2 + bx + c = 0$, then $\sum \alpha = \ldots$ and $\alpha\beta = \ldots$

A) $\frac{b}{a}$ and $-\frac{c}{a}$ B) $-\frac{b}{a}$ and $\frac{c}{a}$ C) $-\frac{b}{a}$ and $-\frac{c}{a}$ D) $\frac{b}{a}$ and $\frac{c}{a}$

25) If α and β are the roots of a quadratic equation $3x^2 - 4x + 7 = 0$, then $\sum \alpha = \ldots$ and $\alpha\beta = \ldots$

A) $\frac{4}{3}$ and $\frac{7}{3}$ B) $-\frac{4}{3}$ and $-\frac{7}{3}$ C) $\frac{4}{3}$ and $-\frac{7}{3}$ D) $-\frac{4}{3}$ and $\frac{7}{3}$

26) If α, β and γ are the roots of a cubic equation $ax^3 + bx^2 + cx + d = 0$, then $\sum \alpha = \alpha+\beta+\gamma = \ldots\ldots$

A) $-\frac{b}{a}$ B) $\frac{c}{a}$ C) $-\frac{d}{a}$ D) None of these

27) If α, β and γ are the roots of a cubic equation $x^3 + px^2 + qx + r = 0$, then $\sum \alpha = \ldots\ldots$

A) -p B) q C) -r D) None of these

28) If α, β and γ are the roots of a cubic equation $ax^3 + bx^2 + cx + d = 0$, then $\sum \alpha\beta = \ldots\ldots$

A) $-\frac{b}{a}$ B) $\frac{c}{a}$ C) $-\frac{d}{a}$ D) None of these

29) If α, β and γ are the roots of a cubic equation $x^3 + px^2 + qx + r = 0$, then $\sum \alpha\beta = \ldots\ldots$

A) - p B) q C) -r D) None of these

30) If α, β and γ are the roots of a cubic equation $ax^3 + bx^2 + cx + d = 0$, then

$\alpha\beta\gamma = \ldots\ldots$

A) $-\frac{b}{a}$ B) $\frac{c}{a}$ C) $-\frac{d}{a}$ D) None of these

31) If α, β and γ are the roots of a cubic equation $x^3 + px^2 + qx + r = 0$, then $\sum \alpha\beta\gamma = \ldots\ldots$

A) - p B) q C) -r D) None of these

32) If α, β, γ and δ are the roots of a biquadratic equation $ax^4 + bx^3 + cx^2 + dx + e = 0$, then $\sum \alpha = \alpha+\beta+\gamma+\delta = \ldots\ldots$

A) $-\frac{b}{a}$ B) $\frac{c}{a}$ C) $-\frac{d}{a}$ D) $\frac{e}{a}$

33) If α, β, γ and δ are the roots of a biquadratic equation $x^4+px^3+qx^2+rx+s = 0$, then $\sum \alpha = \ldots\ldots$

A) -p B) q C) - r D) s

34) If α, β, γ and δ are the roots of a biquadratic equation $ax^4+bx^3+cx^2+dx+e = 0$, then $\sum \alpha\beta = \ldots\ldots$

A) $-\frac{b}{a}$ B) $\frac{c}{a}$ C) $-\frac{d}{a}$ D) $\frac{e}{a}$

35) If α, β, γ and δ are the roots of a biquadratic equation $x^4+px^3+qx^2+rx+s = 0$, then $\sum \alpha\beta = \ldots\ldots$

A) –p B) q C) - r D) s

36) If α, β, γ and δ are the roots of a biquadratic equation $ax^4+bx^3+cx^2+dx+e = 0$, then $\sum \alpha\beta\gamma = \ldots\ldots$

A) $-\frac{b}{a}$ B) $\frac{c}{a}$ C) $-\frac{d}{a}$ D) $\frac{e}{a}$

37) If α, β, γ and δ are the roots of a biquadratic equation $x^4+px^3+qx^2+rx+s = 0$, then $\sum \alpha\beta\gamma = \ldots\ldots$

A) -p B) q C) - r D) s

38) If α, β, γ and δ are the roots of a biquadratic equation $ax^4+bx^3+cx^2+dx+e = 0$, then $\alpha\beta\gamma\delta = \ldots\ldots$

A) $-\frac{b}{a}$ B) $\frac{c}{a}$ C) $-\frac{d}{a}$ D) $\frac{e}{a}$

39) If α, β, γ and δ are the roots of a biquadratic equation $x^4+px^3+qx^2+rx+s = 0$, then $\sum \alpha\beta\gamma\delta = \ldots\ldots$

A) -p B) q C) - r D) s

40) If α, β and γ are the roots of a cubic equation $x^3 - 5x^2 - 16x + 30 = 0$, then $\sum \alpha = \ldots\ldots$

A) 5 B) -5 C) -16 D) None of these

41) If α and β are the roots of equation $x^2 - 5x + 1 = 0$ then $\frac{1}{\alpha} + \frac{1}{\beta} = \ldots\ldots$

A) 5 B) -5 C) -1 D) None of these

42) If α, β, γ are the roots of equation $x^3 - 9x^2 + 14x + 24 = 0$ then $\alpha + \beta + \gamma = \ldots\ldots$

A) 24 B) -9 C) 9 D) None of These

43) If α, β, γ are the roots of equation $x^3 + 9x^2 + 14x + 24 = 0$,

then $\alpha + \beta + \gamma = \ldots\ldots$

A) 24 B) -9 C) 9 D) None of These

44) If α, β, γ are the roots of equation $x^3 - 5x^2 - 16x + 80 = 0$ if sum two roots being equal to zero then $\sum \alpha = \ldots\ldots$

A) 1 B) 5 C) -5 D) None of these

45) If α, β, γ are the roots of equation $x^3 + 2x^2 + 5x - 24 = 0$ then $\sum \alpha\beta =$ ------

A) -5 B) 5 C) -24 D) None of These

46) If α, β, γ and δ are the roots of an equation $x^4 + x^3 + x^2 + x + 1 = 0$, then $\sum \alpha = \ldots\ldots$

A) -1 B) 1 C) 0 D) None of These

47) If α, β, γ and δ are the roots of an equation $x^4 + x^3 + x^2 + x + 1 = 0$, then $\sum \alpha\beta = \ldots\ldots$

A) -1 B) 1 C) 0 D) None of These

48) If α, β, γ and δ are the roots of an equation $x^4 + x^3 + x^2 + x + 1 = 0$, then $\sum \alpha\beta\gamma = \ldots\ldots$

A) -1 B) 1 C) 0 D) None of These

49) If α, β, γ and δ are the roots of an equation $x^4 + x^3 + x^2 + x + 1 = 0$, then $\sum \alpha\beta\gamma\delta = \ldots\ldots$

A) -1 B) 1 C) 0 D) None of These

50) If α, β, γ and δ are the roots of an equation $x^4 - x^3 + x^2 - x + 1 = 0$, then $\sum \alpha = \ldots\ldots$

A) -1 B) 1 C) 0 D) None of These

51) If α, β, γ and δ are the roots of an equation $x^4 - x^3 + x^2 - x + 1 = 0$, then $\sum \alpha\beta = \ldots\ldots$

A) -1 B) 1 C) 0 D) None of These

52) If α, β, γ and δ are the roots of an equation $x^4 - x^3 + x^2 - x + 1 = 0$, then $\sum \alpha\beta\gamma = \ldots\ldots$

A) -1 B) 1 C) 0 D) None of These

53) If α, β, γ and δ are the roots of an equation $x^4 - x^3 + x^2 - x + 1 = 0$, then $\sum \alpha\beta\gamma\delta = \ldots\ldots$

A) -1 B) 1 C) 0 D) None of These

54) The equation $x^4 + 4x^3 - 2x^2 + 2x + 9 = 0$ has two pairs of equal roots then $\alpha + \alpha + \beta + \beta = \ldots\ldots$

A) -4 B) 4 C) -2 D) None of These

55) Solution of equation $x^3 - 3x^2 - 16x + 48 = 0$ is ……

A) {1, 2, 3} B) {3, 4, -4} C) {1, 3, 5} D) None of These

56) Solution of equation $x^3 - 5x^2 - 16x + 80 = 0$ is ……

A) {1, 2, 3} B) {3, 4, -4} C) {5, 4, -4} D) None of These

57) Solution of equation $x^3 - 9x^2 + 23x - 15 = 0$ is ……

A) {1, 3, 5} B) {1, 3, -5} C) {2, 0, 3} D) None of These

58) Solution of equation $x^3 - 7x^2 + 36 = 0$ is ……

A) {0, 1, -1} B) {-2, 3, 6} C) {1, -2, 5} D) None of These

59) Roots $\alpha - \beta, \alpha, \alpha + \beta$ of cubic equation are in …… progression.

A) arithmetic B) geometric C) harmonic D) None of These

60) Roots $\frac{\alpha}{\beta}, \alpha, \alpha\beta$ of cubic equation are in …… progression.

A) arithmetic B) geometric C) harmonic D) None of These

61) An expression in roots which is remain same after interchange of roots is called ……function of roots.

A) symmetric B) linear C) quadratic D) None of These

62) If α, β, γ are the roots of equation $x^3 + px^2 + qx + r = 0$, then $\sum \alpha^2 =$ ……

A) p^2+2q B) $p^2- 2q$ C) 2r D) None of These

63) If α, β, γ are the roots of equation $x^3 - 3x^2 + 4x - 1 = 0$, then $\sum \alpha^2 =$ ……

A) 17 B) 1 C) -2 D) None of These

64) If $\alpha, \beta, \gamma, \delta$ are the roots of equation $x^4 + px^3 + qx^2 + rx + s = 0$, then $\sum \alpha^2 =$ ……

A) p^2+2q B) $p^2- 2q$ C) 2r D) None of These

65) If α, β, γ are the roots of equation $x^3 + px^2 + qx + r = 0$, then $\sum \alpha^2 \beta =$ ……

A) pq+3r B) 3r-pq C) pq-3r D) None of These

66) If α, β, γ are the roots of equation $x^3 - 3x^2 + 4x - 1 = 0$, then $\sum \alpha^2\beta =$ ……

A) -2 B) 1 C) 9 D) None of These

67) If $\alpha, \beta, \gamma, \delta$ are the roots of equation $x^4 + px^3 + qx^2 + rx + s = 0$, then $\sum \alpha^2\beta =$ ……

A) pq+3r B) 3r-pq C) pq-3r D) None of These

68) $\alpha+\beta$, $\alpha^2+\beta^2$, $\frac{1}{\alpha} + \frac{1}{\beta}$, $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ are symmetric functions of two roots α & β is ……

A) True B) False C) May be true or false D) None of These

69) $\alpha-\beta$, $\alpha^2-\beta^2$, $\frac{1}{\alpha} - \frac{1}{\beta}$, $\frac{\alpha}{\beta} - \frac{\beta}{\alpha}$ are symmetric functions of two roots α & β is ……

A) True B) False C) May be true or false D) None of These

70) $\sum \alpha^2 =$ ……

A) $(\sum \alpha)^2 - 2(\sum \alpha\beta)$ B) $(\sum \alpha)^2 + 2(\sum \alpha\beta)$

C) $(\sum \alpha)^2 - \sum \alpha\beta$ D) None of These

==

UNIT-4. THEORY OF EQUATIONS –II

==

The Transformation of Equations:

I) Roots with sign changed:

Let $\alpha_1, \alpha_2, \ldots\ldots, \alpha_n$ are roots of equation $f(x) \equiv a_o x^n + a_1 x^{n-1} + a_2 x^{n-2} + \ldots + a_n = 0$

$\therefore f(x) \equiv a_o (x-\alpha_1)(x-\alpha_2) \ldots\ldots (x-\alpha_n) = 0$

If we replace x by –x, we get,

$f(-x) \equiv a_o (-1)^n (x+\alpha_1)(x+\alpha_2) \ldots\ldots (x+\alpha_n) \ldots\ldots (1)$

As $x + \alpha_i = 0 \Rightarrow x = -\alpha_i$

$\therefore -\alpha_1, -\alpha_2, \ldots\ldots, -\alpha_n$ are the roots of (1).

But $f(-x) = 0$ gives $a_o x^n - a_1 x^{n-1} + a_2 x^{n-2} - \ldots + (-1)^n a_n = 0$ which has roots $-\alpha_1, -\alpha_2, \ldots\ldots, -\alpha_n$.

==

Working Rule: To change the roots of given equation, replace x by –x in the given equation and use $(-x)^n = x^n$ if n is even and $(-x)^n = -x^n$ if n is odd. If highest degree term of this transformed equation is negative, then write the equation by changing signs of each terms.

==

Ex.: Find the equation whose roots are negatives of the roots of $5x^4 + 4x^2 - 7x + 5 = 0$.

Solution: Let $5x^4 + 4x^2 - 7x + 5 = 0$ be the given equation.

Replace x by –x, we get,

$5(-x)^4 + 4(-x)^2 - 7(-x) + 5 = 0$

i.e. $5x^4 + 4x^2 + 7x + 5 = 0$ be the required equation.

==

Ex.: Change the signs of the roots of $3x^8 + 5x^5 - 2x^2 + 4 = 0$.

Solution: Let $3x^8 + 5x^5 - 2x^2 + 4 = 0$ be the given equation.

Replace x by –x, we get,

$3(-x)^8 + 5(-x)^5 - 2(-x)^2 + 4 = 0$

i.e. $3x^8 - 5x^5 - 2x^2 + 4 = 0$ be the required equation.

==

Ex.: Find the equation whose roots are equal in magnitude but opposite in signs of the roots of $x^5 + 4x^3 - 6x^2 + 4x - 7 = 0$. (Mar.2019)

Solution: Let $x^5 + 4x^3 - 6x^2 + 4x - 7 = 0$ be the given equation.

Replace x by –x, we get,

$(-x)^5 + 4(-x)^3 - 6(-x)^2 + 4(-x) - 7 = 0$

i.e. $-x^5 - 4x^3 - 6x^2 - 4x - 7 = 0$

i.e. $x^5 + 4x^3 + 6x^2 + 4x + 7 = 0$ be the required equation.

==

II) Reciprocal Roots: To find the equation whose roots are reciprocals of the roots of given equation.

Let $\alpha_1, \alpha_2, \ldots\ldots, \alpha_n$ are the roots of equation $f(x) \equiv a_o x^n + a_1 x^{n-1} + a_2 x^{n-2} + \ldots + a_n = 0$

$\therefore f(x) \equiv a_o (x-\alpha_1)(x-\alpha_2) \ldots\ldots (x-\alpha_n) = 0$

If we replace x by $\frac{1}{x}$, we get,

$f(\frac{1}{x}) \equiv a_o (\frac{1}{x} - \alpha_1)(\frac{1}{x} - \alpha_2) \ldots\ldots (\frac{1}{x} - \alpha_n) = 0 \ldots\ldots (1)$

As $\frac{1}{x} - \alpha_i = 0 \Rightarrow x = \frac{1}{\alpha_i}$

$\therefore \frac{1}{\alpha_1}, \frac{1}{\alpha_2}, \ldots\ldots, \frac{1}{\alpha_3}$ are the roots of (1).

But $f(\frac{1}{x}) \equiv a_o(\frac{1}{x})^n + a_1(\frac{1}{x})^{n-1} + a_2(\frac{1}{x})^{n-2} + \ldots + a_n = 0$

$\Rightarrow a_0 + a_1x + a_2x^2 + \ldots\ldots + a_nx^n = 0$ i.e. $a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_0 = 0$

be the required equation whose roots are $\frac{1}{\alpha_1}, \frac{1}{\alpha_2}, \ldots\ldots, \frac{1}{\alpha_3}$.

==

Working Rule: Write the given equation in complete form and reverse the order of the coefficients.

==

Ex.: Transform the equation to the equation whose roots are reciprocals of the roots of $3x^2 + 7x - 13 = 0$.

Solution: The complete form of given equation is $3x^2 + 7x - 13 = 0$

By reversing the order of coefficients 3, 7, -13 as -13, 7, 3, we get,

$-13x^2 + 7x + 3 = 0$

i.e. $13x^2 - 7x - 3 = 0$ be the required equation whose roots are reciprocals of the roots of given equation.

==

Ex.: Transform the equation to the equation whose roots are reciprocals of the roots of $2x^5 - 4x^3 + 6x + 7 = 0$

Solution: The complete form of given equation is $2x^5 + 0x^4 - 4x^3 + 0x^2 + 6x + 7 = 0$

By reversing the order of coefficients 2, 0, -4, 0, 6, 7 as 7, 6, 0, -4, 0, 2, we get,

$7x^5 + 6x^4 + 0x^3 - 4x^2 + 0x + 2 = 0$

i.e. $7x^5 + 6x^4 - 4x^2 + 2 = 0$ be the required equation whose roots are reciprocals of the roots of given equation.

==

Ex.: Find the equation whose roots are the reciprocal of the roots.

$x^5 - 4x^3 + 6x^2 - 3x + 2 = 0$ (Mar.2019)

Solution: The complete form of given equation is $x^5 + 0x^4 - 4x^3 + 6x^2 - 3x + 2 = 0$

By reversing the order of coefficients 1, 0, -4, 6, -3, 2 as 2, -3, 6, -4, 0, 1, we get,

$2x^5 - 3x^4 + 6x^3 - 4x^2 + 0x + 1 = 0$

i.e. $2x^5 - 3x^4 + 6x^3 - 4x^2 + 1 = 0$ be the required equation whose roots are reciprocals of the roots of given equation.

==

III) Multiple Roots: To find the equation whose roots are m times the roots of given eq^n.

Let $\alpha_1, \alpha_2, \ldots\ldots, \alpha_n$ are the roots of the given equation

$f(x) \equiv a_ox^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_n = 0$

$\therefore f(x) \equiv a_o (x-\alpha_1)(x-\alpha_2) \ldots\ldots (x-\alpha_n) = 0$

If we replace x by $\frac{x}{m}$, we get,

$f(\frac{x}{m}) \equiv a_o (\frac{x}{m} - \alpha_1)(\frac{x}{m} - \alpha_2) \ldots\ldots (\frac{x}{m} - \alpha_n) = 0 \ldots\ldots (1)$

As $\frac{x}{m} - \alpha_i = 0 \Rightarrow x = m\alpha_i$

$\therefore m\alpha_1,\ m\alpha_2, \ldots\ldots, m\alpha_n$ are the roots of (1).

But $f(\frac{x}{m}) \equiv a_o(\frac{x}{m})^n + a_1(\frac{x}{m})^{n-1} + a_2(\frac{x}{m})^{n-2} + \ldots + a_n = 0$

$\Rightarrow a_o x^n + m a_1 x^{n-1} + m^2 a_2 x^{n-2} + \ldots + m^n a_n = 0$

be the required equation whose roots are $m\alpha_1,\ m\alpha_2, \ldots\ldots, m\alpha_n$.

==

Working Rule: Write the given equation in complete form and multiply each term from 1^{st} to onwards by 1, m, m^2, ……m^n respectively.

Remark: i) To make coefficient of highest degree term 1 of an equation $a_o x^n + a_1 x^{n-1} + a_2 x^{n-2} + \ldots + a_n = 0$, multiply the roots by a_o.

ii) To remove fractional coefficients from given equation, choose the least m such that after multiplying roots by m fractional coefficients removed.

==

Ex.: Find the equation whose roots are three times the roots of $x^3 - 7x^2 + 5x - 1 = 0$.

Solution: The complete form of given equation is $x^3 - 7x^2 + 5x - 1 = 0$

By multiplying each term from 1^{st} to onwards by 1, 3, 3^2, 3^3 respectively, we get,

$1(x^3) + 3(-7x^2) + 3^2(5x) + 3^3(-1) = 0$

i.e. $x^3 - 21x^2 + 45x - 27 = 0$

be the required equation whose roots are three times the roots of given equation.

==

Ex.: Obtain the equation whose roots are three times the roots of $3x^4 - 4x^3 + 4x^2 - 2x + 1 = 0$.

Solution: The complete form of given equation is $3x^4 - 4x^3 + 4x^2 - 2x + 1 = 0$

By multiplying each term from 1^{st} to onwards by 1, 3, 3^2, 3^3, 3^4 respectively, we get,

$1(3x^4) + 3(-4x^3) + 3^2(4x^2) + 3^3(-2x) + 3^4(1) = 0$

i.e. $3x^4 - 12x^3 + 36x^2 - 54x + 81 = 0$

i.e. $x^4 - 4x^3 + 12x^2 - 18x + 27 = 0$ be the required equation whose roots are three times the roots of given equation.

==

Ex.: Transform the equation $x^3 - \frac{1}{2}x^2 + \frac{2}{3}x - 1 = 0$ so that roots will become multiple of the roots of the equation and the fractional coefficient will be removed. (Mar.2019)

Solution: Multiplying the roots of given equation by m, we get transferred equation as,

$x^3 + m(-\frac{1}{2}x^2) + m^2(\frac{2}{3}x) + m^3(-1) = 0$

To remove the fractions we choose m = 6 as a least number,

i.e. $x^3 + 6(-\frac{1}{2}x^2) + 36(\frac{2}{3}x) + 216(-1) = 0$

i.e. $x^3 - 3x^2 + 24x - 216 = 0$ be the required equation.

Ex.: Obtain the equation with multiples of roots of $x^3 - \frac{5}{2}x^2 - \frac{7}{18}x + \frac{1}{108} = 0$ and the fractional coefficient are removed.

Solution: Multiplying the roots of given equation by m, we get transferred equation as,

$x^3 + m(-\frac{5}{2}x^2) + m^2(-\frac{7}{18}x) + m^3(\frac{1}{108}) = 0$

To remove the fractions, we choose m = 6 as a least number,

i.e. $x^3 + 6(-\frac{5}{2}x^2) + 36((-\frac{7}{18}x) + 216(\frac{1}{108}) = 0$

i.e. $x^3 - 15x^2 - 14x + 2 = 0$ be the required equation.

==

Ex.: Transform the equation $5x^3 - \frac{3}{2}x^2 - \frac{3}{4}x + 1 = 0$ to make the roots multiple of the roots of given equation with coefficient of $x^3 = 1$ and removing fractional coefficient.

Solution: The complete form of given equation is $5x^3 - \frac{3}{2}x^2 - \frac{3}{4}x + 1 = 0$.

Dividing by 5, we get,

$x^3 - \frac{3}{10}x^2 - \frac{3}{20}x + \frac{1}{5} = 0$

Multiplying the roots of this equation by m, we get transferred equation as,

$x^3 + m(-\frac{3}{10}x^2) + m^2(-\frac{3}{20}x) + m^3(\frac{1}{5}) = 0$

To remove the fractions, we choose m = 10 as a least number,

i.e. $x^3 + 10(-\frac{3}{10}x^2) + 100(-\frac{3}{20}x) + 1000(\frac{1}{5}) = 0$

i.e. $x^3 - 3x^2 - 15x + 200 = 0$ be the required equation.

==

Ex.: Remove the fractional coefficient from the equation $5x^3 + \frac{3}{2}x^2 + \frac{x}{4} - 2 = 0$ and make the coefficient of leading term 1.

Solution: The complete form of given equation is $5x^3 + \frac{3}{2}x^2 + \frac{x}{4} - 2 = 0$.

Dividing by 5, we get,

$x^3 + \frac{3}{10}x^2 + \frac{1}{20}x - \frac{2}{5} = 0$

Multiplying the roots of this equation by m, we get transferred equation as,

$x^3 + m(\frac{3}{10}x^2) + m^2(\frac{1}{20}x) + m^3(-\frac{2}{5}) = 0$

To remove the fractions, we choose m = 10 as a least number,

i.e. $x^3 + 10(\frac{3}{10}x^2) + 100(\frac{1}{20}x) + 1000(-\frac{2}{5}) = 0$

i.e. $x^3 + 3x^2 + 5x - 400 = 0$ be the required equation.

==

IV) To Diminish or Increase the Roots by h: To obtain an equation whose roots are diminished by h that the roots of given equation.

Let $\alpha_1, \alpha_2, \ldots\ldots, \alpha_n$ are the roots of the given equation

$f(x) \equiv a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_n = 0$

$\therefore f(x) \equiv a_0 (x-\alpha_1)(x-\alpha_2) \ldots\ldots (x-\alpha_n) = 0$

If we replace x by $x + h$, we get,

$f(x+h) \equiv a_o(x+h-\alpha_1)(x+h-\alpha_2)\ldots\ldots(x+h-\alpha_n) = 0 \ldots\ldots (1)$

As $x + h - \alpha_i = 0 \Rightarrow x = \alpha_i - h$

$\therefore \alpha_1$-h, α_2-h, ……, α_n-h are the roots of (1).

But $f(x+h) \equiv a_o(x+h)^n + a_1(x+h)^{n-1} + a_2(x+h)^{n-2} + \ldots + a_n = 0$

$\Rightarrow A_o x^n + A_1 x^{n-1} + A_2 x^{n-2} + \ldots + A_n = 0$ say

be the required equation whose roots are α_1-h, α_2-h, ……, α_n-h.

Where $A_o, A_1, A_2, \ldots, A_n$ are the remainders obtained by dividing (x-h) successively to the quotients by synthetic division.

==

Working Rule: Write the given equation in complete form and use synthetic division to find new coefficients $A_o, A_1, A_2, \ldots, A_n$ by dividing (x-h) successively to the quotients. $A_o x^n + A_1 x^{n-1} + A_2 x^{n-2} + \ldots + A_n = 0$ be the required equation whose roots are diminished by h.

==

Ex.: Find the equation whose roots are the roots of $8x^3 - 4x^2 + 6x - 1 = 0$ each diminished by 2.

Solution: The complete form of given equation is $8x^3 - 4x^2 + 6x - 1 = 0$.

Using synthetic division to divide f(x) and successive quotients by (x-2) as

2	8	-4	6	-1
		16	24	60
	8	12	30	59 = A_3
		16	56	
	8	28	86 = A_2	
		16		
	8	44 = A_1		
	8 = A_0			

The required equation whose roots are diminished by 2 that the roots of given equation is

$A_o x^3 + A_1 x^2 + A_2 x + A_3 = 0$

i.e. $8x^3 + 44x^2 + 86x + 59 = 0$

==

Ex.: Find the equation whose roots are the roots of $3x^3 - 2x^2 + x - 9 = 0$ each diminished by 3.

Solution: The complete form of given equation is $3x^3 - 2x^2 + x - 9 = 0$

Using synthetic division to divide f(x) and successive quotients by (x-3) as

3	3	-2	1	-9
		9	21	66
	3	7	22	57 = A_3
		9	48	
	3	16	70 = A_2	
		9		
	3	25 = A_1		
	3 = A_0			

The required equation whose roots are diminished by 3 that the roots of given equation is

$A_o x^3 + A_1 x^2 + A_2 x + A_3 = 0$

i.e. $3x^3 + 25x^2 + 70x + 57 = 0$

==

Ex.: Find the equation whose roots are the roots of $x^5 + 4x^3 - x^2 + 11 = 0$ each diminished by 3.

Solution: The complete form of given equation is $x^5 + 0x^4 + 4x^3 - x^2 + 0x + 11 = 0$

Using synthetic division to divide f(x) and successive quotients by (x-3) as

3	1	0	4	-1	0	11
		3	9	39	114	342
	1	3	13	38	114	353 = A_5
		3	18	93	393	
	1	6	31	131	507 = A_4	
		3	27	174		
	1	9	58	305 = A_3		
		3	36			
	1	12	94 = A_2			
		3				
	1	15 = A_1				
	1 = A_0					

The required equation whose roots are diminished by 3 that the roots of given equation is

$A_o x^5 + A_1 x^4 + A_2 x^3 + A_3 x^2 + A_4 x + A_5 = 0$

i.e. $x^5 + 15x^4 + 94x^3 + 305x^2 + 507x + 353 = 0$.

==

Ex.: Find the equation whose roots are the roots of $x^4 - x^3 - 10x^2 + 4x + 24 = 0$ increased by 2.

Solution: The complete form of given equation is $x^4 - x^3 - 10x^2 + 4x + 24 = 0$.

Using synthetic division to divide f(x) and successive quotients by (x+2) as

-2	1	-1	-10	4	24
		-2	6	8	-24
	1	-3	-4	12	0 = A_4
		-2	10	-12	
	1	-5	6	0 = A_3	
		-2	14		
	1	-7	20 = A_2		
		-2			
	1	-9 = A_1			
	1 = A_0				

The required equation whose roots are increased by 2 that the roots of given equation is

$A_o x^4 + A_1 x^3 + A_2 x^2 + A_3 x + A_4 = 0$

i.e. $x^4 - 9x^3 + 20x^2 + 0x + 0 = 0$

i.e. $x^4 - 9x^3 + 20x^2 = 0$.

Remark: To remove the second term from the equation $a_o x^n + a_1 x^{n-1} + a_2 x^{n-2} + \ldots + a_n = 0$ diminish the roots by $h = -\frac{a_1}{n a_0}$.

==

Ex.: Remove the second term from the equation $x^4 + 8x^3 + x - 5 = 0$

Solution: The complete form of given equation is $x^4 + 8x^3 + 0x^2 + x - 5 = 0$.

To remove the second term from the given equation, diminish the roots by

$h = -\frac{a_1}{n a_0} = -\frac{8}{4(1)} = -2$

Using synthetic division to divide f(x) and successive quotients by (x+2) as

```
-2 | 1     8     0     1     -5
   |      -2   -12    24    -50
   ----------------------------
   | 1     6   -12    25    -55 = A4
   |      -2    -8    40
   ----------------------
   | 1     4   -20    65 = A3
   |      -2    -4
   ----------------
   | 1     2   -24 = A2
   |      -2
   ----------
   | 1     0 = A1
   |
   ------
   | 1 = A0
```

The required equation whose second term is removed is

$A_o x^4 + A_1 x^3 + A_2 x^2 + A_3 x + A_4 = 0$

i.e. $x^4 + 0x^3 - 24x^2 + 65x - 55 = 0$

i.e. $x^4 - 24x^2 + 65x - 55 = 0$.

==

Ex.: Remove the second term from the equation $x^4 - 12x^3 + 14x - 5 = 0$

Solution: The complete form of given equation is $x^4 - 12x^3 + 0x^2 + 14x - 5 = 0$.

To remove the second term from the given equation diminish the roots by

$h = -\frac{a_1}{n a_0} = -\frac{-12}{4(1)} = 3$

Using synthetic division to divide f(x) and successive quotients by (x-3) as

```
 3 | 1   -12     0     14     -5
   |       3   -27    -81   -201
   -----------------------------
   | 1    -9   -27    -67   -206 = A4
   |       3   -18   -135
   ----------------------
   | 1    -6   -45   -202 = A3
   |       3    -9
   ----------------
   | 1    -3   -54 = A2
   |       3
   ----------
   | 1     0 = A1
   |
   ----------
   | 1 = A0
```

The required equation whose second term is removed is

$A_o x^4 + A_1 x^3 + A_2 x^2 + A_3 x + A_4 = 0$

i.e. $x^4 + 0x^3 - 54x^2 - 202x - 206 = 0$

i.e. $x^4 - 54x^2 - 202x - 206 = 0$.

==

Ex.: Solve the equation $x^3 - 12x^2 + 48x - 72 = 0$ by removing the second term.

Solution: The complete form of given equation is $x^3 - 12x^2 + 48x - 72 = 0$.

To remove the second term from the given equation diminish the roots by

$h = -\frac{a_1}{na_0} = -\frac{-12}{3(1)} = 4$

Using synthetic division to divide f(x) and successive quotients by (x-4) as

```
4 | 1   -12    48    -72
  |       4   -32     64
  ---------------------------
  | 1    -8    16     -8 = A3
  |       4   -16
  ---------------------
  | 1    -4     0 = A2
  |       4
  ---------------
  | 1     0 = A1
  |
  ---------------
  | 1 = A0
```

The required equation whose second term is removed is

$A_o x^3 + A_1 x^2 + A_2 x + A_3 = 0$

i.e. $x^3 + 0x^2 + 0x - 8 = 0$

i.e. $x^3 - 8 = 0$.

$\therefore (x-2)(x^2 + 2x + 4) = 0$

$\therefore x = 2,\ x = \frac{-2\pm\sqrt{4-16}}{2} = -1\pm\sqrt{3}\, i$

be the roots of transferred equation which are less than 4 than the roots of given equation.

$\therefore$ The roots of given equation are $2+4$, $-1+\sqrt{3}\, i + 4$ and $-1-\sqrt{3}\, i + 4$

i.e. 6, $3 + \sqrt{3}\, i$ and $3 - \sqrt{3}\, i$ are the roots of given equation.

==

Ex.: Solve the equation $x^3 + 6x^2 + 12x – 19 = 0$ by removing the second term.

Solution: The complete form of given equation is $x^3 + 6x^2 + 12x – 19 = 0$.

To remove the second term from the given equation diminish the roots by

$h = -\frac{a_1}{na_0} = -\frac{6}{3(1)} = -2$

Using synthetic division to divide f(x) and successive quotients by (x+2) as

```
-2 | 1    6    12    -19
   |     -2    -8     -8
  ---------------------------
   | 1    4     4    -27 = A3
   |     -2    -4
  ---------------------
   | 1    2     0 = A2
   |     -2
  ---------------
   | 1    0 = A1
   |
  ---------------
   | 1 = A0
```

The required equation whose second term is removed is

$A_o x^3 + A_1 x^2 + A_2 x + A_3 = 0$

i.e. $x^3 + 0x^2 + 0x - 27 = 0$

i.e. $x^3 - 27 = 0$.

$\therefore (x-3)(x^2 + 3x + 9) = 0$

$\therefore x = 3,\ x = \frac{-3\pm\sqrt{9-36}}{2} = -\frac{3}{2} \pm \frac{3\sqrt{3}}{2} i$

be the roots of transferred equation which are greater than 2 than the roots of given equation.

$\therefore$ The roots of given equation are 3-2, $-\frac{3}{2} + \frac{3\sqrt{3}}{2} i - 2$ and $-\frac{3}{2} - \frac{3\sqrt{3}}{2} i - 2$

i.e. 1, $-\frac{7}{2} + \frac{3\sqrt{3}}{2} i$ and $-\frac{7}{2} - \frac{3\sqrt{3}}{2} i$ are the roots of given equation.

==

Ex.: Solve the equation $x^4 + 16x^3 + 83x^2 + 152x + 84 = 0$ by removing the second term.

Solution: The complete form of given equation is $x^4 + 16x^3 + 83x^2 + 152x + 84 = 0$.

To remove the second term from the given equation diminish the roots by

$h = -\frac{a_1}{na_0} = -\frac{16}{4(1)} = -4$

Using synthetic division to divide f(x) and successive quotients by (x+4) as

-4	1	16	83	152	84
		-4	-48	-140	-48
	1	12	35	12	$36 = A_4$
		-4	-32	-12	
	1	8	3	$0 = A_3$	
		-4	-16		
	1	4	$-13 = A_2$		
		-4			
	1	$0 = A_1$			
	$1 = A_0$				

The required equation whose second term is removed is

$A_0x^4 + A_1x^3 + A_2x^2 + A_3x + A_4 = 0$

i.e. $x^4 + 0x^3 - 13x^2 + 0x + 36 = 0$

i.e. $x^4 - 13x^2 + 36 = 0$.

$\therefore (x^2 - 4)(x^2 - 9) = 0$

$\therefore x = 2, -2, 3, -3$

are the roots of transferred equation which are greater than 4 than the roots of given equation.

$\therefore$ The roots of given equation are 2-4, -2 – 4, 3 - 4 and -3 - 4

i.e. -2, -6, -1 and -7 are the roots of given equation.

==

Ex.: Solve the equation $x^4 + 20x^3 + 143x^2 + 430x + 462 = 0$ by removing the second term.

Solution: The complete form of given equation is $x^4 + 20x^3 + 143x^2 + 430x + 462 = 0$.

To remove the second term from the given equation diminish the roots by

$h = -\frac{a_1}{na_0} = -\frac{20}{4(1)} = -5$

Using synthetic division to divide f(x) and successive quotients by (x+5) as

-5	1	20	143	430	462
		-5	-75	-340	-450
	1	15	68	90	12 = A_4
		-5	-50	-90	
	1	10	18	0 = A_3	
		-5	-25		
	1	5	-7 = A_2		
		-5			
	1	0 = A_1			
	1 = A_0				

The required equation whose second term is removed is

$A_o x^4 + A_1 x^3 + A_2 x^2 + A_3 x + A_4 = 0$

i.e. $x^4 + 0x^3 - 7x^2 + 0x + 12 = 0$

i.e. $x^4 - 7x^2 + 12 = 0$.

$\therefore (x^2 - 4)(x^2 - 3) = 0$

$\therefore x = 2, -2, \sqrt{3}, -\sqrt{3}$

be the roots of transferred equation which are greater than 5 than the roots of given equation.

$\therefore$ The roots of given equation are 2-5, -2 – 5, $\sqrt{3}$ - 5 and $-\sqrt{3}$ - 5

i.e. -3, -7, $\sqrt{3}$ - 5 and $-\sqrt{3}$ - 5 are the roots of given equation.

==

Cardon's Method of Solving Cubic Equations:

Let $a_o x^3 + a_1 x^2 + a_2 x + a_3 = 0$ …… (1) be the given cubic equation.

First remove the second term by diminishing the roots by $h = -\frac{a_1}{3a_0}$ and then multiply the roots by a_o, we get transformed equation of type say

$z^3 + 3Hz + G = 0$ …… (2)

By Carden's method assume $z = m^{1/3} + n^{1/3}$ be the root of (2).

$\therefore z^3 = m + n + 3m^{1/3}n^{1/3}(m^{1/3} + n^{1/3})$

$\therefore z^3 = m + n + 3(mn)^{1/3}z$

$\therefore z^3 - 3(mn)^{1/3}z - (m + n) = 0$ …… (3)

Equation (2) and (3) are identical.

$\therefore H = -(mn)^{1/3}$ and $G = -(m + n)$

$\therefore mn = -H^3$ and $m + n = -G$

Solving these we get values of m and n, from which we can find $m^{1/3}$ and $n^{1/3}$.

Using these values, $z_1 = m^{1/3} + n^{1/3}$, $z_2 = m^{1/3}\omega + n^{1/3}\omega^2$ and $z_2 = m^{1/3}\omega^2 + n^{1/3}\omega$ gives the roots of (2).

Where $\omega = \frac{-1+i\sqrt{3}}{2}$ and $\omega^2 = \frac{-1-i\sqrt{3}}{2}$ are the cube root of unity.

$\therefore$ The roots of given equation are $x_1 = \frac{z_1}{a_0} + h$, $x_2 = \frac{z_2}{a_0} + h$ and $x_3 = \frac{z_3}{a_0} + h$.

==

Ex.: Solve the equation $x^3 + 6x^2 + 9x + 4 = 0$ using Carden's Method.

Solution: Let $x^3 + 6x^2 + 9x + 4 = 0$ …… (1) be the given equation.

First we remove the second term from given equation by diminishing the roots by

$h = -\frac{a_1}{na_0} = -\frac{6}{3(1)} = -2$

Using synthetic division to divide f(x) and successive quotients by (x+2) as

```
-2 | 1    6    9    4
   |     -2   -8   -2
   ------------------
   | 1    4    1    2 = A3
   |     -2   -4
   -------------
   | 1    2   -3 = A2
   |     -2
   --------
   | 1    0 = A1
   |
   ----
   | 1 = A0
```

The required equation whose second term is removed is

$A_0z^3 + A_1z^2 + A_2z + A_3 = 0$

i.e. $z^3 + 0z^2 - 3z + 2 = 0$

i.e. $z^3 - 3z + 2 = 0$ …… (2)

By Carden's method assume $z = m^{1/3} + n^{1/3}$ be the root of (2).

$\therefore z^3 = m + n + 3m^{1/3}n^{1/3}(m^{1/3} + n^{1/3})$

$\therefore z^3 = m + n + 3(mn)^{1/3}z$

$\therefore z^3 - 3(mn)^{1/3}z - (m + n) = 0$ …… (3)

Equation (2) and (3) are identical.

$\therefore -3(mn)^{1/3} = -3$ and $-(m + n) = 2$

$\therefore (mn)^{1/3} = 1$ and $(m + n) = -2$

$\therefore mn = 1$ and $m + n = -2$

$\therefore m = -1$ and $n = -1$

$\therefore m^{1/3} = -1$ and $n^{1/3} = -1$

Using these values and ω the cube root of unity, we have,

$z_1 = m^{1/3} + n^{1/3} = -1 - 1 = -2,$

$z_2 = m^{1/3}\omega + n^{1/3}\omega^2 = -\omega - \omega^2 = -(\omega + \omega^2) = -(-1) = 1 \quad \because 1 + \omega + \omega^2 = 0$

and $z_2 = m^{1/3}\omega^2 + n^{1/3}\omega = -\omega^2 - \omega = -(\omega^2 + \omega) = -(-1) = 1$

i.e. -2, 1, 1 are the roots of (2).

$\therefore$ The roots of given equation are $x_1 = -2-2 = -4$, $x_2 = 1-2 = -1$ and $x_3 = 1-2 = -1$.

==

Ex.: Solve the equation $x^3 - 3x^2 + 12x + 16 = 0$ using Carden's Method.

Solution: Let $x^3 - 3x^2 + 12x + 16 = 0$ …… (1) be the given equation.

First we remove the second term from given equation by diminishing the roots by

$h = -\frac{a_1}{na_0} = -\frac{-3}{3(1)} = 1$

Using synthetic division to divide f(x) and successive quotients by (x-1) as

```
 1 | 1   -3   12   16
   |      1   -2   10
   ------------------
```

| 1   -2   10 $26 = A_3$
| 1   -1
| 1   -1 $9 = A_2$
| 1
| 1 $0 = A_1$
|
| $1 = A_0$

The required equation whose second term is removed is

$A_oz^3 + A_1z^2 + A_2z + A_3 = 0$

i.e. $z^3 + 0z^2 + 9z + 26 = 0$

i.e. $z^3 + 9z + 26 = 0$ …… (2)

By Carden's method assume $z = m^{1/3} + n^{1/3}$ be the root of (2).

$\therefore z^3 = m + n + 3m^{1/3}n^{1/3}(m^{1/3} + n^{1/3})$

$\therefore z^3 = m + n + 3(mn)^{1/3}z$

$\therefore z^3 - 3(mn)^{1/3}z – (m + n) = 0$ …… (3)

Equation (2) and (3) are identical.

$\therefore - 3(mn)^{1/3}= 9$ and $- (m + n) = 26$

$\therefore (mn)^{1/3}= -3$ and $(m + n) = -26$

$\therefore mn = - 27$ and $m + n = - 26$

$\therefore m = - 27$ and $n = 1$

$\therefore m^{1/3} = - 3$ and $n^{1/3} = 1$

By using cube roots of unity $\omega = \frac{-1+i\sqrt{3}}{2}$ and $\omega^2= \frac{-1-i\sqrt{3}}{2}$, the roots are

$z_1 = m^{1/3}+ n^{1/3}= -3 + 1 = -2,$

$z_2 = m^{1/3}\omega + n^{1/3}\omega^2 = -3(\frac{-1+i\sqrt{3}}{2}) + (\frac{-1-i\sqrt{3}}{2}) = 1-2i\sqrt{3}$ $\quad \because \omega^2 = \frac{-1-i\sqrt{3}}{2}$

and $z_2 = m^{1/3}\omega^2 + n^{1/3} \omega = = - 3(\frac{-1-i\sqrt{3}}{2}) + (\frac{-1+i\sqrt{3}}{2}) = 1+2i\sqrt{3}$

i.e. $-2, 1-2i\sqrt{3}, 1+2i\sqrt{3}$ are the roots of (2).

$\therefore$ The roots of given equation are

$x_1 = -2+1 = -1, x_2 = 1 - 2i\sqrt{3} + 1 = 2 - 2i\sqrt{3}$ and $x_3 = 1+ 2i\sqrt{3} +1 = 2 + 2i\sqrt{3}$

==

Ex.: Solve the equation $x^3 - 15x - 126 = 0$ using Carden's Method.

Solution: Let $x^3 - 15x - 126 = 0$ …… (1) be the given equation with second term absent.

By Carden's method assume $z = m^{1/3} + n^{1/3}$ be the root of (2).

$\therefore z^3 = m + n + 3m^{1/3}n^{1/3}(m^{1/3} + n^{1/3})$

$\therefore z^3 = m + n + 3(mn)^{1/3}z$

$\therefore z^3 - 3(mn)^{1/3}z – (m + n) = 0$ …… (3)

Equation (2) and (3) are identical.

$\therefore - 3(mn)^{1/3}= -15$ and $- (m + n) = -126$

$\therefore (mn)^{1/3}= 5$ and $(m + n) = 126$

$\therefore mn = 125$ and $m + n = 126$

$\therefore m = 125$ and $n = 1$

$\therefore m^{1/3} = 5$ and $n^{1/3} = 1$

By using cube roots of unity $\omega = \frac{-1+i\sqrt{3}}{2}$ and $\omega^2 = \frac{-1-i\sqrt{3}}{2}$, the roots are

$z_1 = m^{1/3} + n^{1/3} = 5 + 1 = 6$,

$z_2 = m^{1/3}\omega + n^{1/3}\omega^2 = 5(\frac{-1+i\sqrt{3}}{2}) + (\frac{-1-i\sqrt{3}}{2}) = -3+2i\sqrt{3}$

and $z_2 = m^{1/3}\omega^2 + n^{1/3}\omega = = 5(\frac{-1-i\sqrt{3}}{2}) + (\frac{-1+i\sqrt{3}}{2}) = -3-2i\sqrt{3}$

i.e. 6, $-3+2i\sqrt{3}$, $-3-2i\sqrt{3}$ are the roots of given equation.

==

Ex.: Solve the equation $x^3 - 18x - 35 = 0$ using Carden's Method.

Solution: Let $x^3 - 18x - 35 = 0$ …… (1) be the given equation with second term absent.

By Carden's method assume $z = m^{1/3} + n^{1/3}$ be the root of (2).

$\therefore z^3 = m + n + 3m^{1/3}n^{1/3}(m^{1/3} + n^{1/3})$

$\therefore z^3 = m + n + 3(mn)^{1/3}z$

$\therefore z^3 - 3(mn)^{1/3}z - (m + n) = 0$ …… (3)

Equation (2) and (3) are identical.

$\therefore -3(mn)^{1/3} = -18$ and $-(m + n) = -35$

$\therefore (mn)^{1/3} = 6$ and $(m + n) = 35$

$\therefore mn = 216$ and $m + n = 35$

$\therefore m = 27$ and $n = 8$

$\therefore m^{1/3} = 3$ and $n^{1/3} = 2$

By using cube roots of unity $\omega = \frac{-1+i\sqrt{3}}{2}$ and $\omega^2 = \frac{-1-i\sqrt{3}}{2}$, the roots are

$z_1 = m^{1/3} + n^{1/3} = 3 + 2 = 5$,

$z_2 = m^{1/3}\omega + n^{1/3}\omega^2 = 3(\frac{-1+i\sqrt{3}}{2}) + 2(\frac{-1-i\sqrt{3}}{2}) = \frac{-5+i\sqrt{3}}{2}$

and $z_2 = m^{1/3}\omega^2 + n^{1/3}\omega = = 3(\frac{-1-i\sqrt{3}}{2}) + 2(\frac{-1+i\sqrt{3}}{2}) = \frac{-5-i\sqrt{3}}{2}$

i.e. 5, $\frac{-5+i\sqrt{3}}{2}$ and $\frac{-5-i\sqrt{3}}{2}$ are the roots of given equation.

==

Ex.: Solve the equation $x^3 - 12x - 65 = 0$ using Carden's Method.

Solution: Let $x^3 - 12x - 65 = 0$ …… (1) be the given equation with second term absent.

By Carden's method assume $z = m^{1/3} + n^{1/3}$ be the root of (2).

$\therefore z^3 = m + n + 3m^{1/3}n^{1/3}(m^{1/3} + n^{1/3})$

$\therefore z^3 = m + n + 3(mn)^{1/3}z$

$\therefore z^3 - 3(mn)^{1/3}z - (m + n) = 0$ …… (3)

Equation (2) and (3) are identical.

$\therefore -3(mn)^{1/3} = -12$ and $-(m + n) = -65$

$\therefore (mn)^{1/3} = 4$ and $(m + n) = 65$

$\therefore mn = 64$ and $m + n = 65$

$\therefore m = 64$ and $n = 1$

$\therefore m^{1/3} = 4$ and $n^{1/3} = 1$

By using cube roots of unity $\omega = \frac{-1+i\sqrt{3}}{2}$ and $\omega^2 = \frac{-1-i\sqrt{3}}{2}$, the roots are

$z_1 = m^{1/3} + n^{1/3} = 4 + 1 = 5$,

$z_2 = m^{1/3}\omega + n^{1/3}\omega^2 = 4(\frac{-1+i\sqrt{3}}{2}) + (\frac{-1-i\sqrt{3}}{2}) = \frac{-5+3i\sqrt{3}}{2}$

and $z_2 = m^{1/3}\omega^2 + n^{1/3}\omega = = 4(\frac{-1-i\sqrt{3}}{2}) + (\frac{-1+i\sqrt{3}}{2}) = \frac{-5-3i\sqrt{3}}{2}$

i.e. 5, $\frac{-5+3i\sqrt{3}}{2}$ and $\frac{-5-3i\sqrt{3}}{2}$ are the roots of given equation.

==

Descarte's Rule of Signs:

1) Descarte's Rule of Signs for Positive Roots of f(x) = 0:

If in a given equation f(x) = 0, number of sign changes from + to – and from – to + starting from first term is k, then an equation f(x) = 0 has at most k positive roots.

i.e. number of positive roots of an equation f(x) = 0 is $\leq$ k.

2) Descarte's Rule of Signs for Negative Roots of f(x) = 0:

If in a equation f(-x) = 0, number of sign changes from + to – and from – to + starting from first term is r, then an equation f(x) = 0 has at most r negative roots.

i.e. number of negative roots of an equation f(x) = 0 is $\leq$ r.

e. g. i) The equation $f(x) \equiv x^3 - 6x^2 + 11x - 6 = 0$ has number of positive roots ≤ 3.

$\because$ Number of sign changes from + to – and from – to + = 3.

The equation $f(x) \equiv x^3 - 6x^2 + 11x - 6 = 0$ has have no negative roots.

$\because f(-x) \equiv -x^3 - 6x^2 - 11x - 6 = 0$ has no sign changes from + to – and from – to +.

ii) The equation $f(x) \equiv x^4 - 9x^2 + 4 = 0$ has number of positive roots ≤ 2

$\because$ Number of sign changes from + to – and from – to + = 2.

The equation $f(x) \equiv x^4 - 9x^2 + 4 = 0$ has number of negative roots ≤ 2.

$\because f(-x) \equiv x^4 - 9x^2 + 4 = 0$ has 2 sign changes from + to – and from – to +.

iii) The equation $f(x) \equiv x^4 + 16x^3 + 83x^2 + 152x + 84 = 0$ has no positive roots.

$\because$ Number of sign changes from + to – and from – to + = 0.

The equation $f(x) \equiv x^4 + 16x^3 + 83x^2 + 152x + 84 = 0$ has number of negative roots ≤ 4

$\because f(-x) \equiv x^4 - 16x^3 + 83x^2 - 152x + 84 = 0$ has 4 sign changes from + to – and from – to +.

iv) The equation $f(x) \equiv x^3 - 12x^2 + 48x - 72 = 0$ has number of positive roots ≤ 3.

$\because$ Number of sign changes from + to – and from – to + = 3.

The equation $f(x) \equiv x^3 - 6x^2 + 11x - 6 = 0$ has have no negative roots.

$\because f(-x) \equiv -x^3 - 12x^2 - 48x - 72 = 0$ has no sign changes from + to – and from – to +.

==

Descarte's Method of Solving Biquadratic Equations:

Let $x^4 + a_2x^2 + a_3x + a_4 = 0$ be the given biquadratic equation in which $a_0 = 1$ and second term is absent, then assume

$x^4 + a_2x^2 + a_3x + a_4 = (x^2 - \lambda x + \mu)(x^2 + \lambda x + \mu')$ ……(2)

Equating the coefficients x^2, x and constant terms, we get,

$\mu + \mu' - \lambda^2 = a_2$; $\lambda(\mu - \mu') = a_3$ and $\mu\mu' = a_4$ …… (3)

To eliminate μ and μ′ from (3) consider

$(\mu + \mu')^2 = (\mu - \mu')^2 + 4\mu\mu'$ i.e. $(\lambda^2 + a_2)^2 = (\frac{a_3}{\lambda})^2 + 4a_4$

which is equation in λ, by inspection find one of the root of it.

Using this root λ, we find μ and μ′. Put these values of μ, μ′ and λ in (2),

we get $x^2 - \lambda x + \mu = 0$ and $x^2 + \lambda x + \mu' = 0$.

Solving we get solution of (2).

Ex.: Solve the equation $x^4 - 5x^2 - 6x - 5 = 0$ by using Descarte's Method.

Solution: Let $x^4 - 5x^2 - 6x - 5 = 0$ …… (1)

be the given biquadratic equation in which $a_0 = 1$ and second term absent.

∴ By Descarte's method assume

$x^4 - 5x^2 - 6x - 5 = (x^2 - \lambda x + \mu)(x^2 + \lambda x + \mu') = 0$ …… (2)

Equating the coefficients x^2, x and constant terms, we get,

$\mu + \mu' - \lambda^2 = -5$; $\lambda(\mu - \mu') = -6$ and $\mu\mu' = -5$ …… (3)

To eliminate μ and μ′ from (3) consider

$(\mu + \mu')^2 = (\mu - \mu')^2 + 4\mu\mu'$

$\therefore (\lambda^2 - 5)^2 = (\frac{-6}{\lambda})^2 + 4(-5)$

$\therefore \lambda^4 - 10\lambda^2 + 25 = \frac{36}{\lambda^2} - 20$

$\therefore \lambda^6 - 10\lambda^4 + 25\lambda^2 = 36 - 20\lambda^2$

$\therefore \lambda^6 - 10\lambda^4 + 45\lambda^2 - 36 = 0$ …… (4)

Here sum of all coefficients = 0 ⇒ λ = 1 is one of root of (4).

Putting λ = 1 in (3), we get,

$\mu + \mu' = -4$ and $\mu - \mu' = -6$

Adding we get, $2\mu = -10 \Rightarrow \mu = -5 \Rightarrow -5 + \mu' = -4 \Rightarrow \mu' = 1$

find μ and μ′ . Putting these values of μ, μ′ and λ in (2)

$x^4 - 5x^2 - 6x - 5 = (x^2 - x - 5)(x^2 + x + 1) = 0$

$\therefore x^2 - x - 5 = 0$ or $x^2 + x + 1 = 0$

$\therefore\ x = \frac{1 \pm \sqrt{1+20}}{2}$ or $x = \frac{-1 \pm \sqrt{1-4}}{2}$

$\therefore\ x = \frac{1 \pm \sqrt{21}}{2}$ or $x = \frac{-1 \pm i\sqrt{3}}{2}$

be the required roots of given equation.

Ex.: Solve the equation $x^4 - 8x^2 - 24x + 7 = 0$ by using Descarte's Method.

Solution: Let $x^4 - 8x^2 - 24x + 7 = 0$ …… (1)

be the given biquadratic equation in which $a_0 = 1$ and second term absent.

∴ By Descarte's method assume

$x^4 - 8x^2 - 24x + 7 = (x^2 - \lambda x + \mu)(x^2 + \lambda x + \mu') = 0$ …… (2)

Equating the coefficients x^2, x and constant terms, we get,

$\mu + \mu' - \lambda^2 = -8$; $\lambda(\mu - \mu') = -24$ and $\mu\mu' = 7$ …… (3)

To eliminate μ and μ′ from (3) consider

$(\mu + \mu')^2 = (\mu - \mu')^2 + 4\mu\mu'$

$\therefore (\lambda^2 - 8)^2 = (\frac{-24}{\lambda})^2 + 4(7)$

$\therefore \lambda^4 - 16\lambda^2 + 64 = \frac{576}{\lambda^2} + 28$

$\therefore \lambda^6 - 16\lambda^4 + 64\lambda^2 = 576 + 28\lambda^2$

$\therefore \lambda^6 - 16\lambda^4 + 36\lambda^2 - 576 = 0$ …… (4)

By inspection λ = 4 is one of root of (4).

Putting λ = 4 in (3), we get,

$\mu + \mu' = 8$ and $\mu - \mu' = -6$

Adding we get, $2\mu = 2 \Rightarrow \mu = 1 \Rightarrow 1 + \mu' = 8 \Rightarrow \mu' = 7$

find μ and μ′ . Putting these values of μ, μ′ and λ in (2)

$x^4 - 8x^2 - 24x + 7 = (x^2 - 4x + 1)(x^2 + 4x + 7) = 0$

$\therefore x^2 - 4x + 1 = 0$ or $x^2 + 4x + 7 = 0$

$\therefore\ x = \frac{4 \pm \sqrt{16-4}}{2}$ or $x = \frac{-4 \pm \sqrt{16-28}}{2}$

$\therefore\ x = \frac{4 \pm 2\sqrt{3}}{2}$ or $x = \frac{-4 \pm 2i\sqrt{3}}{2}$

$\therefore\ x = 2 \pm \sqrt{3}$ or $x = -2 \pm i\sqrt{3}$ be the required roots of given equation.

==

Ex.: Solve $x^4 - 3x^2 - 6x - 2 = 0$ by Descarte's Method.

Solution: Let $x^4 - 3x^2 - 6x - 2 = 0$ …… (1)

be the given biquadratic equation in which $a_0 = 1$ and second term absent.

∴ By Descarte's method assume

$x^4 - 3x^2 - 6x - 2 = (x^2 - \lambda x + \mu)(x^2 + \lambda x + \mu') = 0$ …… (2)

Equating the coefficients x^2, x and constant terms, we get,

$\mu + \mu' - \lambda^2 = -3$; $\lambda(\mu - \mu') = -6$ and $\mu\mu' = -2$ …… (3)

To eliminate μ and μ′ from (3) consider

$(\mu + \mu')^2 = (\mu - \mu')^2 + 4\mu\mu'$

$\therefore (\lambda^2 - 3)^2 = (\frac{-6}{\lambda})^2 + 4(-2)$

$\therefore \lambda^4 - 6\lambda^2 + 9 = \frac{36}{\lambda^2} - 8$

$\therefore \lambda^6 - 6\lambda^4 + 9\lambda^2 = 36 - 8\lambda^2$

$\therefore \lambda^6 - 6\lambda^4 + 17\lambda^2 - 36 = 0$ …… (4)

By inspection λ = 2 is one of root of (4).

Putting λ = 2 in (3), we get,

$\mu + \mu' = 1$ and $\mu - \mu' = -3$

Adding we get, $2\mu = -2 \Rightarrow \mu = -1 \Rightarrow -1 + \mu' = 1 \Rightarrow \mu' = 2$

find μ and μ′ . Putting these values of μ, μ′ and λ in (2)

$x^4 - 3x^2 - 6x - 2 = (x^2 - 2x - 1)(x^2 + 2x + 2) = 0$

$\therefore x^2 - 2x - 1 = 0$ or $x^2 + 2x + 2 = 0$

$\therefore\ x = \frac{2\pm\sqrt{4+4}}{2}$ or $x = \frac{-2\pm\sqrt{4-8}}{2}$

$\therefore\ x = \frac{2\pm2\sqrt{2}}{2}$ or $x = \frac{-2\pm2i}{2}$

$\therefore\ x = 1 \pm \sqrt{2}$ or $x = -1 \pm i$ be the required roots of given equation.

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MULTIPLE CHOICE QUESTIONS [MCQ'S]

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1) If $\alpha_1, \alpha_2, \ldots\ldots, \alpha_n$ are the roots of equation $f(x) = 0$ then $-\alpha_1, -\alpha_2, \ldots\ldots, -\alpha_n$ are the roots, of equation ……

A) $f(x) = 0$ B) $-f(x) = 0$ C) $f(-x) = 0$ D) None of these

2) The equation whose roots are negatives of the roots of $5x^4 + 4x^2 - 7x + 5 = 0$ is …

A) $-5x^4 - 4x^2 + 7x - 5 = 0$ B) $5x^4 + 4x^2 + 7x + 5 = 0$

C) $5x^4 + 4x^2 - 7x - 5 = 0$ D) None of these

3) By changing the signs of the roots of equation $3x^8 + 5x^5 - 2x^2 + 4 = 0$, the required equation is …

A) $3x^8 - 5x^5 - 2x^2 + 4 = 0$ B) $3x^8 + 5x^5 - 2x^2 - 4 = 0$

C) $3x^8 + 5x^5 + 2x^2 + 4 = 0$ D) None of these

4) The equation whose roots are negatives of the roots of $x^6 + 5x^3 - 7x^2 + 4x - 8 = 0$, is …

A) $x^6 - 5x^3 + 7x^2 - 4x - 8 = 0$ B) $x^6 - 5x^3 - 7x^2 - 4x - 8 = 0$

C) $x^6 + 5x^3 + 7x^2 + 4x + 8 = 0$ D) None of these

5) The equation whose roots are equal in magnitude but opposite in signs of the roots of $x^5 + 4x^3 - 6x^2 + 4x - 7 = 0$ is …

A) $x^5 + 4x^3 + 6x^2 + 4x - 7 = 0$ B) $x^5 - 4x^3 - 6x^2 - 4x - 7 = 0$

C) $x^5 + 4x^3 + 6x^2 + 4x + 7 = 0$ D) None of these

6) If $\alpha_1, \alpha_2, \ldots\ldots, \alpha_n$ are the roots of equation $a_ox^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_n = 0$, then the equation whose roots are reciprocals of the roots of given equation is …

A) $a_ox^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_n = 0$ B) $x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_n = 0$

C) $a_o + a_1x + a_2x^2 + \ldots + a_{n-1}x^{n-1} + a_nx^n = 0$ D) None of these

7) The equation whose roots are reciprocals of the roots of $3x^2 + 7x - 13 = 0$ is …

A) $3x^2 - 7x + 13 = 0$ B) $13x^2 - 7x - 3 = 0$

C) $3x^2 - 7x - 13 = 0$ D) None of these

8) The equation whose roots are reciprocals of the roots of $x^3 + 5x^2 - 7x + 8 = 0$ is …

A) $8x^3 + 5x^2 - 7x + 1 = 0$ B) $x^3 - 5x^2 - 7x - 8 = 0$

C) $8x^3 - 7x^2 + 5x + 1 = 0$ D) None of these

9) The equation whose roots are reciprocals of the roots of $3x^4 + 4x^3 – 7x^2 + 5x - 1 = 0$ is …

A) $x^4 - 5x^3 + 7x^2 - 4x - 3 = 0$ B) $x^4 + 5x^3 + 7x^2 + 4x + 3 = 0$

C) $3x^4 - 4x^3 – 7x^2 - 5x - 1 = 0$ D) None of these

10) The equation whose roots are reciprocals of the roots of $2x^5 - 4x^3 + 6x + 7 = 0$ is …

A) $7x^5 + 6x^4 - 4x^2 + 2 = 0$ B) $7x^5 + 6x^4 - 4x^2 + x + 2 = 0$

C) $7x^5 + 6x^4 + 4x^2 + 2 = 0$ D) None of these

11) If $\alpha_1, \alpha_2, \ldots\ldots, \alpha_n$ are the roots of equation $a_ox^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_n = 0$, then the equation whose roots are m times the roots of given equation is …

A) $a_ox^n + ma_1x^{n-1} + m^2a_2x^{n-2} + \ldots + m^na_n = 0$ B) $x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_n = 0$

C) $a_o + a_1x + a_2x^2 + \ldots + a_{n-1}x^{n-1} + a_nx^n = 0$ D) None of these

12) If we multiply the roots of equation $x^3 - 3x^2 - 6x + 8 = 0$ by 2, then the transformed equation will be …

A) $x^3 - 6x^2 - 12x + 16 = 0$ B) $2x^3 - 6x^2 - 12x + 16 = 0$

C) $x^3 - 6x^2 - 24x + 64 = 0$ D) $2x^3 - 6x^2 - 24x + 64 = 0$

13) The equation whose roots are three times the roots of $x^3 - 7x^2 + 5x - 1 = 0$ is …

A) $x^3 - 21x^2 + 45x - 27 = 0$ B) $3x^3 - 21x^2 + 15x - 3 = 0$

C) $x^3 - 21x^2 + 45x + 27 = 0$ D) None of these

14) The equation whose roots are three times the roots of $3x^4 - 4x^3 + 4x^2 - 2x + 1 = 0$ is …

A) $9x^4 - 12x^3 + 12x^2 - 6x + 3 = 0$ B) $x^4 - 4x^3 + 12x^2 - 18x + 27 = 0$

C) $x^4 - 4x^3 + 4x^2 - 2x + 1 = 0$ D) None of these

15) The equation obtained from the equation $x^3 - \frac{1}{2}x^2 + \frac{2}{3}x - 1 = 0$ by removing the fractional coefficients is ……

A) $x^3 - 3x^2 + 4x - 6 = 0$ B) $x^3 - x^2 + 2x - 1 = 0$

C) $x^3 - 3x^2 + 24x - 216 = 0$ D) None of these

16) The equation obtained from the equation $x^3 - \frac{2}{3}x^2 + \frac{1}{2}x - 7 = 0$ by removing the fractional coefficients is ……

A) $x^3 - 4x^2 + 18x - 1512 = 0$ B) $x^3 - 2x^2 + x - 7 = 0$

C) $x^3 - 4x^2 + 3x - 42 = 0$ D) None of these

17) The equation obtained from the equation $x^3 - \frac{5}{2}x^2 - \frac{7}{18}x + \frac{1}{108} = 0$ by removing the fractional coefficients is ……

A) $x^3 - 5x^2 - 7x + 1 = 0$ B) $x^3 - 15x^2 - 14x + 2 = 0$

C) $x^3 - 2x^2 - 18x + 108 = 0$ D) None of these

18) To remove the second term from the equation $a_ox^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_n = 0$, the roots should be diminished by h = …

A) $-\frac{a_1}{na_o}$ B) $\frac{a_1}{na_o}$ C) $-\frac{a_1}{a_o}$ D) None of these

19) To remove the second term from the equation $x^4 + 8x^3 + x^2 + x + 3 = 0$, the roots should be diminished by …

A) 2 B) -2 C) 4 D) 5

20) To remove the second term from the equation $x^4 + 8x^3 + x - 5 = 0$, the roots should be diminished by …

A) 2 B) -2 C) 4 D) 5

21) To remove the second term from the equation $x^4 - 8x^3 + x^2 - x + 3 = 0$, the roots should be diminished by …

A) 2 B) -2 C) -4 D) 4

22) To remove the second term from the equation $x^4 + 16x^3 + 83x^2 + 152x + 84 = 0$, the roots should be diminished by …

A) 2 B) -2 C) -4 D) 4

23) To remove the second term from the equation $x^4 - 8x^3 + x^2 - x + 3 = 0$, the roots should be diminished by …

A) 1 B) 2 C) 3 D) None of these

24) To remove the second term from the equation $x^3 - 12x^2 + 48x - 72 = 0$, the roots should be diminished by h =…

A) 2 B) 4 C) -4 D) None of these

25) To remove the second term from the equation $x^3 + 6x^2 + 9x + 4 = 0$, the roots should be increased by …

A) -2 B) 2 C) 6 D) -6

26) By Descarte's rule of signs for positive roots the equation, $f(x) \equiv x^3 - 6x^2 + 11x - 6 = 0$ has number of positive roots …

A) = 3 B) ≤ 3 C) ≥ 3 D) None of these

27) By Descarte's rule of signs for negative roots the equation, $f(x) \equiv x^3 - 6x^2 + 11x - 6 = 0$ has…… negative roots.

A) no B) 2 C) 3 D) None of these

28) By Descarte's rule of signs for positive roots the equation $f(x) \equiv x^4 - 9x^2 + 4 = 0$ has number of positive roots ……

A) =2 B) ≤ 2 C) ≥ 2 D) None of these

29) Number of negative roots of $f(x) \equiv x^4 - 9x^2 + 4 = 0$ is …

A) =2 B) ≤ 2 C) ≥ 2 D) None of these

30) Number of positive roots of the equation $f(x) \equiv x^4 + 16x^3 + 83x^2 + 152x + 84 = 0$ is …

A) 0 B) ≤ 2 C) ≥ 2 D) None of these

31) Number of negative roots of the equation $f(x) \equiv x^4 + 16x^3 + 83x^2 + 152x + 84 = 0$ is …

A) 4 B) ≤ 4 C) ≥ 4 D) None of these

32) Number of positive roots of the equation $f(x) \equiv x^3 - 12x^2 + 48x - 72 = 0$ is …

A) ≤ 1 B) ≤ 2 C) ≤ 3 D) None of these

33) Number of negative roots of the equation $f(x) \equiv x^3 - 12x^2 + 48x - 72 = 0$ is …

A) 0 B) ≤ 1 C) ≤ 2 D) None of these

34) By Carden's method to solve the cubic equation $a_o x^3 + a_1 x^2 + a_2 x + a_3 = 0$, first we remove second term by diminishing the roots by h = ….

A) $-\frac{a_1}{3a_o}$ B) $\frac{a_1}{3a_o}$ C) $-\frac{a_1}{a_o}$ D) None of these

35) By Carden's method to solve the cubic equation $z^3 + 3Hz^2 + G = 0$, assume one of root z = …… .

A) m+n B) $m^{1/3} + n^{1/3}$ C) $m^{1/3} - n^{1/3}$ D) None of these

36) If ω is the cube root of unity, then ω = ……

A) $\frac{-1+i\sqrt{3}}{2}$ B) $\frac{-1-i\sqrt{3}}{2}$ C) 0 D) None of these

37) If ω is the cube root of unity, then ω^2 = ……

A) $\frac{-1+i\sqrt{3}}{2}$ B) $\frac{-1-i\sqrt{3}}{2}$ C) 0 D) None of these

38) If ω is the cube root of unity, then $1+\omega+\omega^2 = \ldots\ldots$

A) 1 B) −1 C) 0 D) None of these

39) If $x^4 + a_2x^2 + a_3x + a_4 = (x^2 - \lambda x + \mu)(x^2 + \lambda x + \mu')$, then $a_2 = \ldots\ldots$

A) $\mu + \mu' - \lambda^2$ B) $\lambda(\mu - \mu')$ C) $\mu\mu'$ D) None of these

40) If $x^4 + a_2x^2 + a_3x + a_4 = (x^2 - \lambda x + \mu)(x^2 + \lambda x + \mu')$, then $a_3 = \ldots\ldots$

A) $\mu + \mu' - \lambda^2$ B) $\lambda(\mu - \mu')$ C) $\mu\mu'$ D) None of these

41) If $x^4 + a_2x^2 + a_3x + a_4 = (x^2 - \lambda x + \mu)(x^2 + \lambda x + \mu')$, then $a_4 = \ldots\ldots$

A) $\mu + \mu' - \lambda^2$ B) $\lambda(\mu - \mu')$ C) $\mu\mu'$ D) None of these

42) If $x^4 - 5x^2 - 6x - 5 = (x^2 - \lambda x + \mu)(x^2 + \lambda x + \mu')$, then $\mu + \mu' - \lambda^2 = \ldots\ldots$

A) 1 B) −5 C) -6 D) None of these

43) If $x^4 - 5x^2 - 6x - 5 = (x^2 - \lambda x + \mu)(x^2 + \lambda x + \mu')$, then $\lambda(\mu - \mu') = \ldots\ldots$

A) 1 B) −5 C) -6 D) None of these

44) If $x^4 - 5x^2 - 6x - 5 = (x^2 - \lambda x + \mu)(x^2 + \lambda x + \mu')$, then $\mu\mu' = \ldots\ldots$

A) 1 B) −5 C) -6 D) None of these

45) If $x^4 - 8x^2 - 24x + 7 = (x^2 - \lambda x + \mu)(x^2 + \lambda x + \mu')$, then $\mu + \mu' - \lambda^2 = \ldots\ldots$

A) -8 B) -24 C) 7 D) None of these

46) If $x^4 - 8x^2 - 24x + 7 = (x^2 - \lambda x + \mu)(x^2 + \lambda x + \mu')$, then $\lambda(\mu - \mu') = \ldots\ldots$

A) -8 B) -24 C) 7 D) None of these

47) If $x^4 - 8x^2 - 24x + 7 = (x^2 - \lambda x + \mu)(x^2 + \lambda x + \mu')$, then $\mu\mu' = \ldots\ldots$

A) -8 B) -24 C) 7 D) None of these

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॥ अंतरी पेटवू ज्ञानज्योत ॥

विद्यापीठ गीत

मंत्र असो हा एकच हृदयी 'जीवन म्हणजे ज्ञान'
ज्ञानामधूनी मिळो मुक्ती अन मुक्तीमधूनी ज्ञान ॥धृ ॥
कला, ज्ञान, विज्ञान, संस्कृती साधू पुरूषार्थ
साफल्यास्तव सदा 'अंतरी पेटवू ज्ञानज्योत'
मंगल पावन चराचरातून स्त्रवते अक्षय ज्ञान ॥१ ॥
उत्तम विद्या, परम शक्ति ही आमुची ध्येयासक्ती
शील, एकता, चारित्र्यावर सदैव आमुची भक्ती
सत्य शिवाचे मंदिर सुंदर, विद्यापीठ महान ॥२ ॥
समता, ममता, स्वातंत्र्याचे नांदो जगी नाते,
आत्मबलाने होऊ आम्ही आमुचे भाग्यविधाते,
ज्ञानप्रभुची लाभो करूणा आणि पायसदान ॥३ ॥

— कै.प्रा. राजा महाजन

THE NATIONAL INTERGRATION PLEDGE

"I solemnly pledge to work with dedication to preserve and strengthen the freedom and integrity of the nation.

I further affirm that I shall never resort to violence and that all differences and disputes relating to religion, language, region or other political or economic grievance should be settled by peaceful and constitutional means."